



Holyoke Public Schools
Mathematics Curriculum Map
Grade 4

Factors, Multiples and Arrays

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Curriculum Maps

GOALS:

1. To ensure that students are exposed to a rigorous curriculum in every school and every grade.
2. To have consistent instruction and assessment district wide.
3. To prepare students for the MCAS test.
4. To explain what is expected to be covered in each CMP or Investigations Unit.

EXPECTATIONS:

The district's expectation is for students to successfully meet the Massachusetts Mathematics Standards. In order to help facilitate this, teachers are required to follow the curriculum maps. The successful implementation of these maps requires teachers to thoroughly read each lesson in the TE and work through the project and problems in the map and the text prior to planning their lessons. Work should be kept in the binder with the curriculum map. Working through the math is an essential part of lesson planning, as it helps the teacher to better understand the concept being taught and the students' possible misunderstandings.

FEEDBACK TO STUDENTS:

Feedback needs to happen daily in the classroom. There are many ways to give feedback. Conferencing, observations, questions asked during your opening, work time and closing are all forms of feedback.

MAP COMPONENTS:

1. GENERAL PROBING QUESTIONS
2. UNIT SPECIFIC PROBING QUESTIONS
3. GOALS OF UNIT, CONTENT STANDARDS, & PERFORMANCE STANDARDS
4. PROJECT- to be done at end of unit and kept in the portfolio.
 - STUDENT MASTER – for project
5. INVESTIGATIONS:
 - NOTEBOOK - includes: 3 Ring Binder, Bound Notebook, Portfolio
 - ACCOUNTABLE TALK – using probing questions
6. ON-DEMAND ASSESSMENTS - to be done during teaching of unit.
 - STUDENT MASTERS- for on-demand assessments.

Mathematics

Evidence of Learning Artifacts

Artifact	K - 1	2 - 5	6 - 8
<i>3 Ring Binder (3R)*</i>	<ul style="list-style-type: none"> ○ Student Work¹ 	<ul style="list-style-type: none"> ○ Vocabulary ○ Student sheets¹ <p style="text-align: center;"><u>All work should be dated and listed by investigation</u></p>	<ul style="list-style-type: none"> ○ Math books ○ Vocabulary ○ Core Problems¹ ○ Lab sheets <p style="text-align: center;"><u>All work should be dated and listed by investigation</u></p>
<i>Marble Notebook (MNB)</i>	<ul style="list-style-type: none"> ○ Journal entries² 	<ul style="list-style-type: none"> ○ Table of Contents ○ Problem of the day ○ Journal entries ○ Class work <p style="text-align: center;"><u>All work should be dated and listed by investigation in the Table of Contents</u></p>	<ul style="list-style-type: none"> ○ Table of Contents ○ Work time ○ Journal entries <p style="text-align: center;"><u>All work should be dated and listed by investigation in the Table of Contents</u></p>
<i>Portfolio³ (P)</i>	<ul style="list-style-type: none"> ○ On-demand tasks ○ Projects ○ Teacher anecdotal notes 	<ul style="list-style-type: none"> ○ On-demand tasks ○ Reflections ○ Projects <p style="text-align: center;"><u>All work should be dated and listed by investigation</u></p>	<ul style="list-style-type: none"> ○ On-demand tasks ○ Reflections ○ Projects <p style="text-align: center;"><u>All work should be dated and listed by investigation</u></p>

* Folders may be used in place of binders for these grade levels

¹ Send home at the end of each unit

² Use grade level math journals

³ All documents should be kept for the entire year

Factors, Multiples, and Arrays Probing Questions for Accountable Talk

As students progress through this unit, they should be asked the following questions to assess their knowledge about problem situations that involve factors, multiples and arrays.

- Can you represent that in a different way?
- What is the relationship between those problems?
- How else could you have solved that problem?
- Can you use other factors to help you solve that problem?
- Can you name another problem that has different factors and the same product?

Ten Minute Math

Today's Number: Session 1.1

Quick Images: Session 2.1

Counting Around the Class: Session 3.1

Ten Minute Math activities offer practice and review of key concepts at each grade level. After their initial introduction, these short activities, designed to take no longer than 10 minutes, support and balance the in-depth work of each curriculum unit.

Implementing Investigations in Grade 4: Please review pages 23 -24, 28 -30 and 33 -34 for 3 Ten Minute Math activities in this unit.

Additional Probing Questions for Accountable Talk

The teacher's role in probing for understanding is to ask questions that will:

- Clarify student understanding
- Get at the objective of the lesson
- Go deeper into the mathematics
- Uncover misconceptions and misunderstandings
- Compare and contrast

The students' role is to be an active participant by:

- Explaining their strategies
- Asking clarifying questions to teacher and other students
- Being active listeners
- Using the language of mathematics

When probing for understanding the teacher and students can use one or more of these suggested questions:

- Why are you using $<$ $>$?
- What are the ways you could $<$ $>$?
- What else do you know?
- How do you know that?
- Can you show that?
- What convention did you use here?
- What can you do if you do not know?
- What standard does this work apply to?
- Is this always true?
- How does this connect to other mathematics we have learned?
- What is the same and what are the differences between $<$ $>$?
- Can you back that up?
- Where is the math in your sketch?
- What does the answer mean?
- Does the answer make sense?
- Could you have used another operation to solve this task?
- Can you give examples?
- Can you say it another way?
- What's the math?
- Tell me about the task in your own words?
- What are you trying to find?
- How did you make your estimate?
- Will your answer be an over-estimate or an under-estimate? Why?
- I noticed that you used $<...>$ to help you understand the task. Can you show us what you did and tell us how it helped you?
- Where do you see $<$ $>$ in your $<$ model, diagram, number line, chart, etc. $>$?
- How can we see $<$ $>$ in your $<$ model, diagram, number line, chart, etc. $>$?
- You have used a representation that is different from others that I've seen. Can you show us your $<$ model, diagram, number line, chart, etc. $>$, and tell us how it helped you?
- How did you decide to solve the task? Why did you choose that method?
- Did you try any method that didn't work?
 - Tell us what you tried, and why it didn't work

Goals, Content Standards, & Performance Standards

Unit Goals:

- Use known multiplication combinations to find the product of any multiplication combination up to 12×12 .
- Use arrays, pictures or models of groups, and story contexts to represent multiplications situations.
- Find the factors of 2-digit numbers.

Math Content Standards:

4.N.7 Recognize classes (in particular, odds, evens; factors or multiples of a given number; and squares) to which a number may belong, and identify the numbers in those classes. Use these in the solution of problems. λ

4.N.8 Select, use, and explain various meanings and models of multiplication and division of whole numbers. Understand and use the inverse relationship between the two operations. λ

4.N.9 Select, use, and explain the commutative, associative, and identity properties of operations on whole numbers in problem situations, e.g., $37 \times 46 = 46 \times 37$, $(5 \times 7) \times 2 = 5 \times (7 \times 2)$. λ

4.N.11 Know multiplication facts through 12×12 and related division facts. Use these facts to solve related multiplication problems and compute related problems, e.g., 3×5 is related to 30×50 , 300×5 , and 30×500 . σ

4.P.3 Determine values of variables in simple equations, e.g., $4106 - \nabla = 37$, $5 = \mu + 3$, and $\square - \mu = 3$. λ

Performance Standards:

M1a Adds subtracts multiplies and divides whole numbers with and without calculators.

M3d Uses letters, boxes, or any symbols to stand for any number, measured quantity or object in simple situations with concrete materials.

M4a Given the basic statement of a problem situation, the student: Uses strategies, such as using manipulatives or drawing sketches, to model problems.

M5b The student makes the choices involved in planning and carrying out a solution; that is, the student: makes connections among concepts in order to solve problems.

M5c The students move beyond a particular problem by making connections, extensions, and/or generalizations; for example, the student: explains a pattern that can be used in similar situations.

UNIT: FACTORS, MULTIPLES, AND ARRAYS

End-of-Unit Project

GRADE: 4

End-of-Unit Project (P)

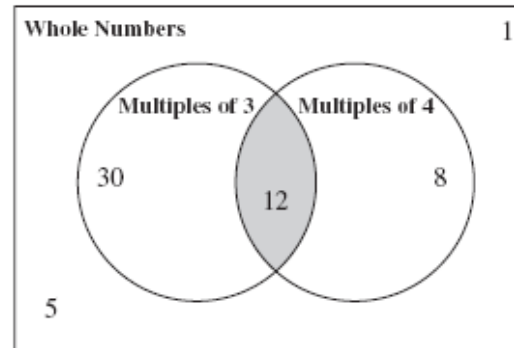
Student work should be placed in **portfolio (P)**.

The project is the culminating assessment which will allow students to apply what they learned about multiplication, division, and reasoning. It is written in MCAS form to give students the experience of answering an open-response question.

10 The diagram below shows how Henry is grouping some numbers.

- One circle has multiples of 3.
- The other circle has multiples of 4.
- The shaded space is for numbers that are multiples of both 3 and 4.
- Numbers that are not a multiple of 3 or 4 go in the space outside the circles.

Henry has already placed the numbers 1, 5, 8, 12, and 30 in the correct spaces in the diagram.



- a. Make a copy of the diagram in your Student Answer Booklet. Write each of the numbers shown below in the correct space in your diagram. Explain how you know where each number belongs.

24 16 27 14 32

- b. Identify one more number that goes in each of the four spaces in the diagram. Be sure to tell the space in the diagram where each number belongs. Explain how you know the numbers you used are placed correctly.

UNIT: FACTORS, MULTIPLES, AND ARRAYS

Investigation 1 (1.1 – 1.5)

DAYS: 5 - 6

GRADE: 4

<p>Evidence of Learning Artifacts</p> <p>Journal and Reflection questions should be posted and referred to at the beginning of the appropriate <i>Investigation</i>.</p> <p>Journal and Reflection entries need to be done in class as part of the closure and assessment.</p>	<p>(3R) – 3 ring binder; (MNB) – marble notebook; (P) – portfolio</p> <p><i>Vocabulary</i> – multiplication, array, dimension (3R)</p> <p><i>Work Time</i> – Student Activity Sheets 1 - 14 (3R)</p> <p><i>Journal Entries</i> – (MNB)</p> <p>Inv. 1.1 What is an array? How are arrays related to multiplication?</p> <p>Inv. 1.2 How are arrays related to area? How are area, multiplication, and arrays related?</p> <p>Inv. 1.3 List all the factors of 30. Explain how you know you have listed all the factors.</p> <p>Inv. 1.4 Explain how you could use simpler multiplication facts to solve more difficult facts.</p> <p>Inv. 1.5 List the strategies you can use to help you solve multiplication facts you don't know.</p> <p><i>Reflection</i> – Describe the operation of multiplication to a second grader. (P)</p>
<p>Accountable Talk</p> <p>To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.</p>	<p><i>As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:</i></p> <p>How did you know that?</p> <p>How can you use ...?</p> <p>Can you show another way?</p> <p>What convention did you use?</p> <p><i>These are some recommended questions that you might use. Others can be found be found at the beginning of the map and on the probing question sheet in the district mathematics guide.</i></p>

UNIT: FACTORS, MULTIPLES, AND ARRAYS

Investigation 2 (2.1 – 2.5)

DAYS: 5

GRADE: 4

<p>Evidence of Learning Artifacts</p> <p>Journal and Reflection questions should be posted and referred to at the beginning of the appropriate <i>Investigation</i>.</p> <p>Journal and Reflection entries need to be done in class as part of the closure and assessment.</p>	<p>(3R) – 3 ring binder; (MNB) –marble notebook; (P) – portfolio</p> <p><i>Vocabulary:</i> factor (3R)</p> <p><i>Work Time:</i> Student Activity Sheets 15 – 24 (3R)</p> <p><i>Journal Entries</i> – (MNB)</p> <p>Inv. 2.1 Name a known fact that could help you solve 7×8. and explain how this fact helped you.</p> <p>Inv. 2.2 None</p> <p>Inv. 2.3 Describe the difference between a factor and a multiple.</p> <p>Inv. 2.4 Describe your strategy for playing Multiple Turn Over.</p> <p>Inv. 2.5 How can Multiplication combinations help you solve unknown multiplication facts.</p> <p><i>Reflection</i> – Explain at least two ways using array cards can help you learn multiplication facts. (P)</p>
<p>Accountable Talk</p> <p>To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.</p>	<p><i>As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:</i></p> <p>How did you know...?</p> <p>How can you justify...?</p> <p>Does your answer make sense?</p> <p>Can you draw me a diagram?</p> <p><i>These are some recommended questions that you might use. Others can be found at the beginning of the map and on the probing question sheet in the district mathematics guide.</i></p>

UNIT: FACTORS, MULTIPLES, AND ARRAYS

Investigation 3 (3.1 – 3.4)

DAYS: 4

GRADE: 4

<p>Evidence of Learning Artifacts</p> <p>Journal and Reflection questions should be posted and referred to at the beginning of the appropriate <i>Investigation</i>.</p> <p>Journal and Reflection entries need to be done in class as part of the closure and assessment.</p>	<p>(3R) – 3 ring binder; (MNB) – marble notebook; (P) – portfolio</p> <p><i>Vocabulary</i> – Reinforce previous vocabulary (3R)</p> <p><i>Work Time</i> – Student Activity sheets 27 - 43 (3R)</p> <p><i>Journal Entries</i> – (MNB)</p> <p>Inv. 3.1 Is 32 a factor of 200? Explain how you know.</p> <p>Inv. 3.2 How can you use factors of 100 to help you find factors of 300?</p> <p>Inv. 3.3 Are the factors of 12 the same as the factors of 24? How do you know?</p> <p>Inv. 3.4 None due to assessment</p> <p><i>Reflection</i> – Explain how you would you find all of the factors of 80? (P)</p>
<p>Accountable Talk</p> <p>To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.</p>	<p><i>As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:</i></p> <p>What is your strategy?</p> <p>How could you check that is correct?</p> <p>Could you begin with a step?</p> <p>Does make sense given the information in the problem?</p> <p><i>These are some recommended questions that you might use. Others can be found be found at the beginning of the map and on the probing question sheet in the district mathematics guide.</i></p>

End-of-Unit Project

Student work should be placed in **portfolio (P)**.

The project is the culminating assessment which will allow students to apply what they learned about factors and multiples. It is written in MCAS form to give students the experience of answering an open-response question.

NAME: _____

DATE: _____

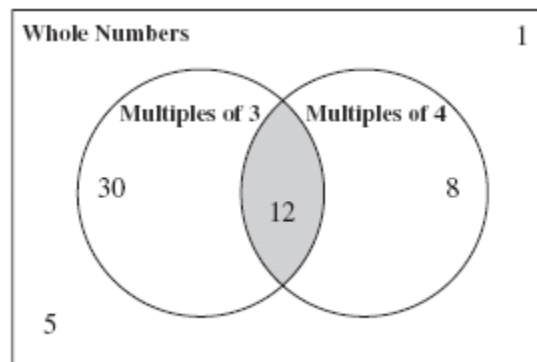
End-of-Unit Project

- **BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.**
- **Show all work (diagrams, tables, and computations) on your answer sheet.**
- **If you do the work in your head, explain in writing how you did the work.**

The diagram below shows how Henry is grouping some numbers.

- One circle has multiples of 3.
- The other circle has multiples of 4.
- The shaded space is for numbers that are multiples of both 3 and 4.
- Numbers that are not a multiple of 3 or 4 go in the space outside the circles.

Henry has already placed the numbers 1, 5, 8, 12, and 30 in the correct spaces in the diagram.



- a. Make a copy of the diagram in your Student Answer Booklet. Write each of the numbers shown below in the correct space in your diagram. Explain how you know where each number belongs.

24 16 27 14 32

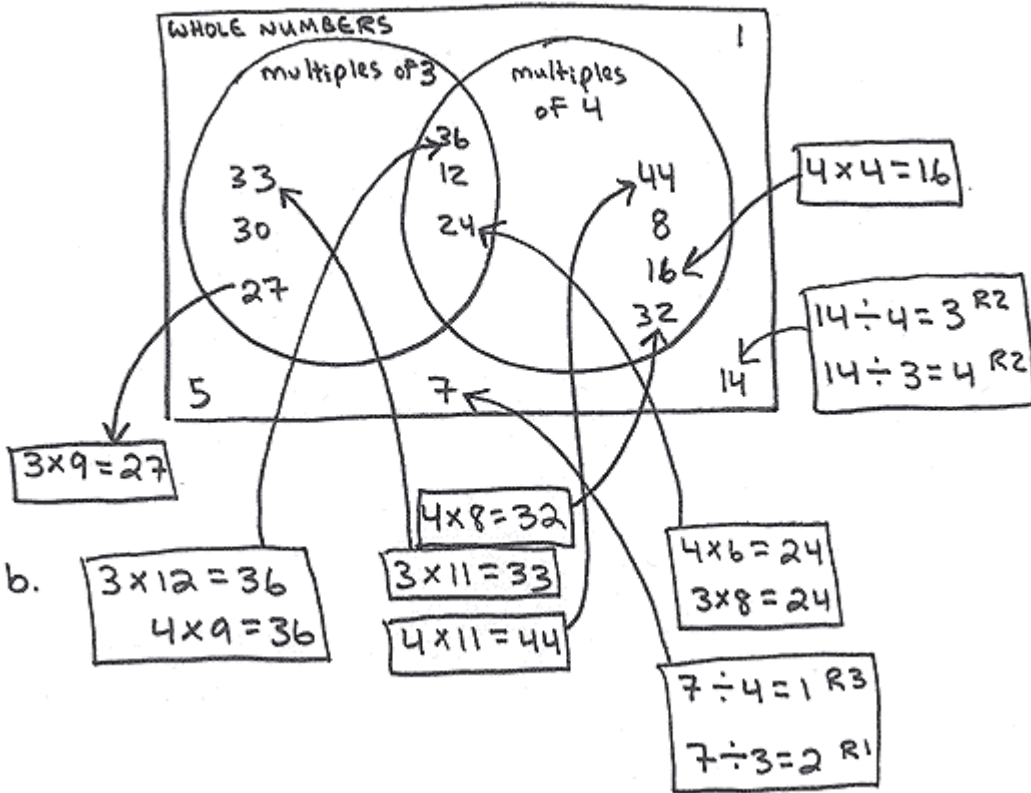
- b. Identify one more number that goes in **each** of the four spaces in the diagram. Be sure to tell the space in the diagram where each number belongs. Explain how you know the numbers you used are placed correctly.

Scoring Guide and Sample Student Work

Score	Description
<u>4</u>	The student response demonstrates an exemplary understanding of the Number Sense and Operations concepts involved in recognizing and identifying multiples of a given number.
<u>3</u>	The student response demonstrates a good understanding of the Number Sense and Operations concepts involved in recognizing and identifying multiples of a given number. Although there is significant evidence that the student was able to recognize and apply the concepts involved, some aspect of the response is flawed. As a result the response merits 3 points.
<u>2</u>	The student response demonstrates a fair understanding of the Number Sense and Operations concepts involved in recognizing and identifying multiples of a given number. While some aspects of the task are completed correctly, others are not. The mixed evidence provided by the student merits 2 points.
<u>1</u>	The student response demonstrates a minimal understanding of the Number Sense and Operations concepts involved in recognizing and identifying multiples of a given number.
<u>0</u>	The student response contains insufficient evidence of an understanding of the Number Sense and Operations concepts involved in recognizing and identifying multiples of a given number to merit any points.

2004 MCAS Grade 4 Mathematics
Question 10 - Score Point 4

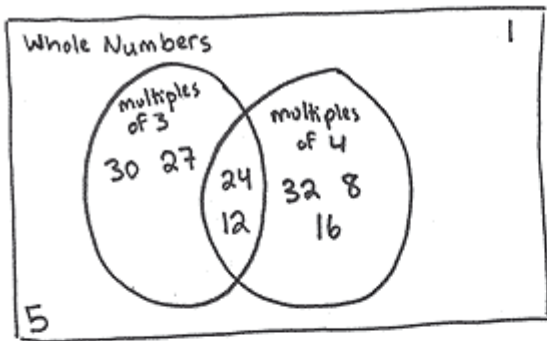
a.



b.

2004 MCAS Grade 4 Mathematics
Question 10 - Score Point 3

a.

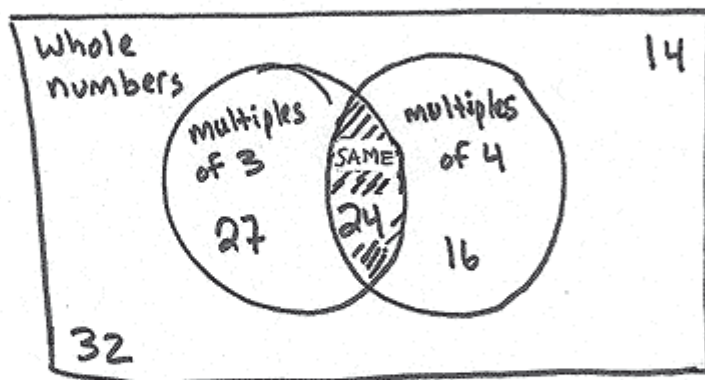


I knew that 30 and 27 were multiples of three by using multiplication. I knew that 32 and 16 went in the group with multiples by also using multiplication. Also using multiplication I was able to group 24 and 12. I didn't put number fourteen anywhere because fourteen isn't a multiple of 3 or 4.

b. The number eighteen goes in the multiples of 3 I used multiplication again to identify where it goes. The number 20 goes in the 4 multiples using multiplication. Lastly, 36 goes in both using multiplication.

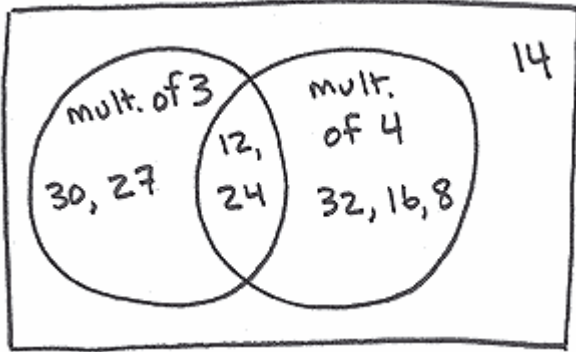
2004 MCAS Grade 4 Mathematics
Question 10 - Score Point 2

- A.) ¹ 24 is a mutiple of 3 and 4
² 16 is a multiple of 4.
³ 27 is a multiple of 3.
⁴ 32 and ⁵ 14 are a whole
number.



- B.) ¹ 36 can go in multiples of 4.
It is a multiple of 4.
² 9 can go in multiples of 3.
It is a multiple of three.
³ 12 can go in the shaded area because
it is a multiple of 3 and 4.
⁴ and ⁵. I don't know.

2004 MCAS Grade 4 Mathematics
Question 10 - Score Point 1



On-Demand Tasks

(To be filed in portfolio)

Factors, Multiples, and Arrays

In class individualized On-Demand tasks assess knowledge of mathematical facts, operations, concepts, and skills, and their efficient application to problem solving. The results of these different forms of assessment provide rich profiles of students' achievements in mathematics and serve as the basis for identifying curricula and instructional approaches to best develop their talents.

UNIT: FACTORS, MULTIPLES, AND ARRAYS

On-Demand Assessments

GRADE: 4

<p>On-Demand Assessments (P)</p> <p><u>Factors, Multiples, and Arrays</u> Investigations</p> <p>In class individualized On-Demand Assessments assess knowledge of mathematical facts, operations, concepts, and skills, and their efficient application to problem solving. The results of these different forms of assessment provide rich profiles of students' achievements in mathematics and serve as the basis for identifying curricula and instructional approaches to best develop their talents.</p>	<p><u>Inv. 1:</u> Resource Binder: Session 1.5, M32* Assessment Checklist, M31**</p> <p><u>Inv. 2:</u> Resource Binder: Session 2.5, M51</p> <p><u>Inv. 3:</u> Resource Binder: Session 3.4, M55-M56**</p> <p>*Assessment Checklists should be kept with tracking sheets.</p> <p>**Please refer to the section in the Teacher's Unit Guide entitled, "Professional Development" for examples of student work for each assessment</p>
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Name _____

Date _____

**Factors, Multiples, and Arrays**

Assessment: Multiplication Combinations

$8 \times 5 =$	$4 \times 7 =$	$3 \times 9 =$
$7 \times 9 =$	$3 \times 12 =$	$6 \times 4 =$
$3 \times 6 =$	$9 \times 4 =$	$7 \times 6 =$
$12 \times 4 =$	$5 \times 7 =$	$8 \times 8 =$
$9 \times 9 =$	$6 \times 6 =$	$5 \times 6 =$
$11 \times 6 =$	$8 \times 9 =$	$6 \times 12 =$
$3 \times 8 =$	$5 \times 12 =$	$8 \times 6 =$
$5 \times 9 =$	$3 \times 7 =$	$7 \times 8 =$
$7 \times 10 =$	$8 \times 4 =$	$7 \times 7 =$
$9 \times 6 =$	$12 \times 12 =$	$8 \times 12 =$

Session 2.5

Unit 1

M51

Name _____

Date _____

Factors, Multiples, and Arrays



End-of-Unit Assessment (page 1 of 2)

Problem 1

A. Solve this multiplication combination.

$$6 \times 9 =$$

Did you use another multiplication combination to help you get the answer? If you did, explain what combination you used and how it helped you find the product of 6×9 .

B. Draw a picture of either arrays, objects, or cubes to show that your answer is correct.

C. Write a story to go with the problem 6×9 .

Name _____

Date _____



Factors, Multiples, and Arrays

End-of-Unit Assessment (page 2 of 2)

Problem 2

You have 36 cans of juice.

A. Show all the ways you can arrange these cans into arrays. You may either draw arrays in the space below or use grid paper.

B. List all the factors of 36.

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M56 Unit 1

Session 3.4



Holyoke Public Schools 2007 - 2008 Mathematics Scoring Rubric

Score point 4:

The response shows a comprehensive understanding of the mathematical concept(s) and/or procedures embodied in the task(s). It indicates that the student has completed the task(s) correctly, using mathematically sound procedures. It contains clear, complete explanations and/or adequate work required.

Score point 3:

The response shows a general understanding of the mathematical concept(s) and/or procedures embodied in the task(s). It indicates that the student has completed the task(s), using mathematically sound procedures. It contains complete explanations and/or adequate work required.

Score point 2:

The response shows a basic understanding of the mathematical concept(s) and/or procedures embodied in the task(s). It addresses most aspects of the task(s), using mathematically sound procedures. It may contain a correct solution but provides incomplete procedures, reasoning and/or explanations. It may reflect some misunderstandings of the underlying mathematical concepts and/or procedures.

Score point 1:

The response shows a minimal understanding of the mathematical concepts and/or procedures embodied in the task(s). It addresses some elements of the task(s) correctly but reaches an inadequate solution and/or provides reasoning that is faulty or incomplete. It exhibits multiple flaws related to a misunderstanding of important aspects of the task(s), misuse of mathematical procedures, or faulty mathematical reasoning. It reflects a lack of essential understanding of the underlying mathematical concepts. It may contain a correct numerical answer but the required work is not provided.

Score point 0:

The response is completely incorrect, irrelevant, or incoherent, or contains a correct response arrived at using an obviously incorrect procedure.

NOTES