Holyoke Public Schools
Mathematics Curriculum Map
Grade 4

Multiple Towers and Division Stories

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Curriculum Maps

GOALS:

1. To ensure that students are exposed to a rigorous curriculum in every school and every grade.
2. To have consistent instruction and assessment district wide.
3. To prepare students for the MCAS test.
4. To explain what is expected to be covered in each CMP or Investigations Unit.

EXPECTATIONS:

The district's expectation is for students to successfully meet the Massachusetts Mathematics Standards. In order to help facilitate this, teachers are required to follow the curriculum maps. The successful implementation of these maps requires teachers to thoroughly read each lesson in the TE and work through the project and problems in the map and the text prior to planning their lessons. Work should be kept in the binder with the curriculum map. Working through the math is an essential part of lesson planning, as it helps the teacher to better understand the concept being taught and the students' possible misunderstandings.

FEEDBACK TO STUDENTS:

Feedback needs to happen daily in the classroom. There are many ways to give feedback. Conferencing, observations, questions asked during your opening, work time and closing are all forms of feedback.

MAP COMPONENTS:

1. GENERAL PROBING QUESTIONS
2. UNIT SPECIFIC PROBING QUESTIONS
3. GOALS OF UNIT, CONTENT STANDARDS, & PERFORMANCE STANDARDS
4. PROJECT- to be done at end of unit and kept in the portfolio.
    - STUDENT MASTER – for project
5. INVESTIGATIONS:
    - NOTEBOOK - includes: 3 Ring Binder, Bound Notebook, Portfolio
    - ACCOUNTABLE TALK – using probing questions
5. ON-DEMAND ASSESSMENTS - to be done during teaching of unit.
    - STUDENT MASTERS- for on-demand assessments.

Mathematics
Evidence of Learning Artifacts

Artifact	K - 1	2 - 5	6 - 8
3 Ring Binder (3R)*	o Student Work[1]	o Vocabulary o Student sheets[1] ***<u>All work should be dated and listed by investigation</u>***	o Math books o Vocabulary o Core Problems[1] o Lab sheets ***<u>All work should be dated and listed by investigation</u>***
Marble Notebook (MNB)	o Journal entries[2]	o Table of Contents o Problem of the day o Journal entries o Class work ***<u>All work should be dated and listed by investigation in the Table of Contents</u>***	o Table of Contents o Work time o Journal entries ***<u>All work should be dated and listed by investigation in the Table of Contents</u>***
Portfolio[3] (P)	o On-demand tasks o Projects o Teacher anecdotal notes	o On-demand tasks o Reflections o Projects ***<u>All work should be dated and listed by investigation</u>***	o On-demand tasks o Reflections o Projects ***<u>All work should be dated and listed by investigation</u>***

[*] *Folders may be used in place of binders for these grade levels*
[1] *Send home at the end of each unit*
[2] *Use grade level math journals*
[3] *All documents should be kept for the entire year*

Multiple Towers and Division Stories
Probing Questions for Accountable Talk

As students progress through this unit, they should be asked the following questions to assess their knowledge about problem situations that involve multiplication and division.

- How can you show the relationship between the operations ?
- Is there another strategy you could have used ?
- How can you show that a different way?
- Will that strategy always work?
- How is your strategy similar/different to other strategies shared ?
- Which strategy do you think is the most efficient? Why?

Ten Minute Math

Continue from previous units
Quick Images
Counting Around the Class

Ten Minute Math activities offer practice and review of key concepts at each grade level. After their initial introduction, these short activities, designed to take no longer than 10 minutes, support and balance the in-depth work of each curriculum unit.

Implementing Investigations in Grade 4: Please review pages 23-24, 28-30 for 2 Ten Minute Math activities in this unit.

Additional Probing Questions for Accountable Talk

The teacher's role in probing for understanding is to ask questions that will:

- Clarify student understanding
- Get at the objective of the lesson
- Go deeper into the mathematics
- Uncover misconceptions and misunderstandings
- Compare and contrast

The students' role is to be an active participant by:

- Explaining their strategies
- Asking clarifying questions to teacher and other students
- Being active listeners
- Using the language of mathematics

When probing for understanding the teacher and students can use one or more of these suggested questions:

- Why are you using < >?
- What are the ways you could < >?
- What else do you know?
- How do you know that?
- Can you show that?
- What convention did you use here?
- What can you do if you do not know?
- What standard does this work apply to?
- Is this always true?
- How does this connect to other mathematics we have learned?
- What is the same and what are the differences between < >?
- Can you back that up?
- Where is the math in your sketch?
- What does the answer mean?
- Does the answer make sense?
- Could you have used another operation to solve this task?
- Can you give examples?
- Can you say it another way?
- What's the math?
- Tell me about the task in your own words?
- What are you trying to find?
- How did you make your estimate?
- Will your answer be an over-estimate or an under-estimate? Why?
- I noticed that you used <….> to help you understand the task. Can you show us what you did and tell us how it helped you?
- Where do you see < > in your <model, diagram, number line, chart, etc.>?
- How can we see < > in your <model, diagram, number line, chart, etc.>?
- You have used a representation that is different from others that I've seen. Can you show us your <model, diagram, number line, chart, etc.>, and tell us how it helped you?
- How did you decide to solve the task? Why did you choose that method?
- Did you try any method that didn't work?
  - Tell us what you tried.
  - Why didn't it work?

Goals, Content Standards, & Performance Standards

Unit Goals:

- Multiply 2-digit numbers by 1-digit and small 2-digit numbers (e.g., 12, 15, 20), using strategies that involve breaking the numbers apart.
- Solve division problems (2-digit and small 3-digit numbers divided by 1-digit numbers), including some that result in a remainder.
- Use story problems, pictures, or concrete models to represent division situations.
- Multiply by 10 and multiples of 10.
- Demonstrate fluency with multiplication combinations up to 12 x 12.

Math Content Standards:

(4.N.8) Select, use, and explain various meanings and models of multiplication and division of whole numbers. Understand and use the inverse relationship between the two operations.

(4.N.9) Select, use, explain the commutative, associative, identity properties of operations on whole numbers in problem situations, e.g., 37 x 46 = 46 x 37, (5 x 7) x 2 = 5 x (7 x 2)

(4.N.10) Select and use appropriate operations(addition, subtraction, multiplication, division) to solve problems, including those involving money.

(4.N.11) Know multiplication facts through 12X12 and related division facts. Use these facts to solve related multiplication problems.

(4.N.12) Add and subtract (up to 5-digit numbers) and multiply (up to 3 digits by 2 digits) accurately and efficiently.

(4.N.13) Divide up to a three-digit whole number with a single digit divisor (with or without remainders) accurately and efficiently. Interpret any remainders.

(4.N.15) Demonstrate in the classroom an understanding of and the ability to use the conventional algorithms for division of up to 3-digit whole number with a single-digit divisor (with or without remainders).

(4.P.1) Create, describe, extend and explain symbolic (geometric) and numeric patterns, including multiplication patterns like 3, 30, 300, 3000

(4.P.3) Determine values of variables in simple operations, e.g., 4106 - __ = 37, 5 = __ + 3 __

Performance Standards:

M1b. Uses and understands the inverse relationships between multiplication and division, and to determine unknown quantities and equations.

M5a. The student formulates and solves a variety of meaningful problems and extracts pertinent information from situations.

M5b. Uses and invents a variety of approaches and understands and evaluates those of others. Student invokes problem solving strategies such as illustrating with sense-making sketches to clarify situations. Student determines when it is helpful to break a problem into simpler parts.

End-of-Unit Project

GRADE: 4

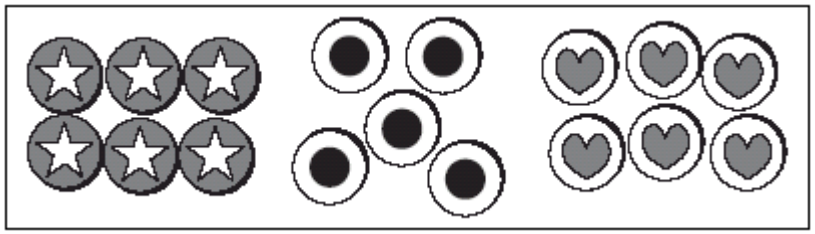
End-of-Unit Project (P)

Student work should be placed in **portfolio (P)**.

The project is the culminating assessment which will allow students to apply what they learned about the use of multiplication, division and reasoning. It is written in MCAS form to give students the experience of answering an open-response question.

2004, Mathematics - Grade 4
Question 27: Open-Response
Reporting Category: Patterns, Relations, and Algebra
Standard(s): 4.P.5

The 17 tokens shown below are used to play a game. Each kind of token is worth a different number of points.



The picture below shows two clues about the number of points each kind of token is worth.

a. If each is worth 20 points, how many points is each worth? Show your work or explain how you got your answer.
b. If 4 friends play the game, they will divide the 17 tokens into 4 groups so that each friend's group of tokens is worth the same number of points. What is the total number of points each friend's group of tokens is worth? Show your work or explain how you got your answer.

UNIT: MULTIPLE TOWERS AND DIVISION STORIES

Investigation 1 (1.1 – 1.5) DAYS: 5

GRADE: 4

Evidence of Learning Artifacts Journal and Reflection questions should be posted and referred to at the beginning of the appropriate *Investigation*. Journal and Reflection entries need to be done in class as part of the closure and assessment.	**(3R) – 3 ring binder; (MNB) – marble notebook; (P) – portfolio** ***Vocabulary*** – multiplication, array, factor, product, equation, (3R) ***Work Time*** **-** Student Activity Sheets 1 - 15 (3R) ***Journal Entries*** – (MNB) **Inv. 1.1** Describe how to break apart larger numbers to help solve multiplication problems. **Inv. 1.2** How can you combine two or three small arrays to make a larger array? **Inv. 1.3** Explain the strategy you used when playing Small Array/Big Array. **Inv. 1.4** Describe the steps you would make to solve 22 x 9. **Inv. 1.5** No journal entry due to assessment. ***Reflection*** – Write a story problem that represents 32 x 12. Explain how you know your problem calls for multiplication.(P)
Accountable Talk To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.	*As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:* How did you know that? How can you use …? Can you show another way? What convention did you use? *These are some recommended questions that you might use. Others can be found be found at the beginning of the map and on the probing question sheet in the district mathematics guide.*

UNIT: MULTIPLE TOWERS AND DIVISION STORIES
Investigation 2 (2.1 – 2.6) DAYS: 6

GRADE: 4

Evidence of Learning Artifacts Journal and Reflection questions should be posted and referred to at the beginning of the appropriate *Investigation*. Journal and Reflection entries need to be done in class as part of the closure and assessment.	**(3R) – 3 ring binder; (MNB) –marble notebook; (P) – portfolio** ***Vocabulary*** – division, remainder, factor, inverse operations, divisor, dividend, quotient (3R) ***Work Time –*** Student Activity Sheets 18 - 35 (3R) ***Journal Entries*** – (MNB) **Inv. 2.1** Describe at least two strategies you used to solve division problems. **Inv. 2.2** When grouping for a division problem, how do you know when you are finished? **Inv. 2.3** In a division story problem in which there is a remainder. Explain what you could do with the remainder. **Inv. 2.4** How can you use multiplication factors to help you solve a division problem? **Inv. 2.5** Explain the relationship between multiplication and division **Inv. 2.6** None due to assessment ***Reflection*** – Explain the difference between sharing and grouping division problems using pictures, numbers and/or words. (P)
Accountable Talk To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.	*As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:* How did you know…? How can you justify…? Does your answer make sense? Can you draw me a diagram? *These are some recommended questions that you might use. Others can be found be found at the beginning of the map and on the probing question sheet in the district mathematics guide.*

UNIT: MULTIPLE TOWERS AND DIVISION STORIES
Investigation 3 (3.1 – 3.4) DAYS: 4

GRADE: 4

Evidence of Learning Artifacts Journal and Reflection questions should be posted and referred to at the beginning of the appropriate *Investigation*. Journal and Reflection entries need to be done in class as part of the closure and assessment.	**(3R) – 3 ring binder; (MNB) – marble notebook; (P) – portfolio** ***Vocabulary*** – multiple, factor, combinations (3R) ***Work Time –*** Student Activity Sheets 39 – 51 (3R) ***Journal Entries*** – (MNB) **Inv. 3.1** What are the differences between a multiple and a factor? **Inv. 3.2** What patterns do you notice when multiplying by 10? **Inv. 3.3** Explain why it works to solve a problem like 3x40 by first thinking of it as 3x4. **Inv. 3.4** When multiplying by 10, what does it mean to "add a zero"? ***Reflection*** – Create a story problem using 13 x 25. Explain your strategy for solving this problem. . (P)
Accountable Talk To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.	*As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:* What is your strategy? How could you check that ….. is correct? Could you begin with a different step? Does …. make sense given the information in the problem? *These are some recommended questions that you might use. Others can be found be found at the beginning of the map and on the probing question sheet in the district mathematics guide.*

UNIT: MULTIPLE TOWERS AND DIVISION STORIES
Investigation 4 (4.1 - 4.5) DAYS: 5

GRADE: 4

Evidence of Learning Artifacts Journal and Reflection questions should be posted and referred to at the beginning of the appropriate *Investigation*. Journal and Reflection entries need to be done in class as part of the closure and assessment.	**(3R) – 3 ring binder; (MNB) – marble notebook; (P) – portfolio** ***Vocabulary*** – doubled, halved, (3R) ***Work Time –*** Student Activity Sheets 52 – 66 (3R) ***Journal Entries*** – (MNB) **Inv. 4.1** What is the relationship between the following two problems: **8 x 14** **16 x 7** **Inv. 4.2** Make a list of cluster problems that might help others to solve **24 x 7.** **Inv. 4.3** Explain how you know whether the following equation is true or not. **15 x 50 = 5 x 150** **Inv. 4.4** Describe the strategy you are most comfortable using when solving 2-digit multiplication problems. **Inv. 4.5** None due to assessment ***Reflection*** – Show at least **two** ways to solve the following related problems: **5 x 7** **5 x 70** **50 x 70** (P)
Accountable Talk To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.	*As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:* Is there a way to write ….? Does …. help you find the answer? *These are some recommended questions that you might use. Others can be found be found at the beginning of the map and on the probing question sheet in the district mathematics guide.*

End-of-Unit Project

Student work should be placed in **portfolio (P)**.

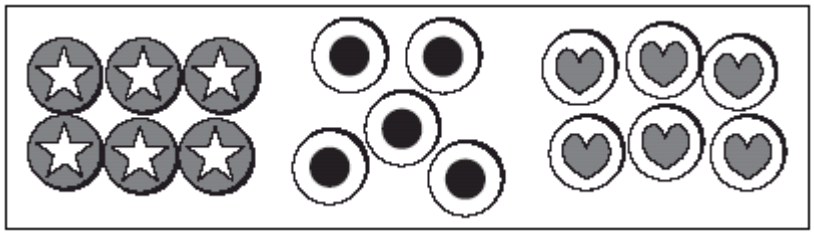
The project is the culminating assessment which will allow students to apply what they learned about multiplication, division and reasoning. It is written in MCAS form to give students the experience of answering an open-response question.

End-of-Unit Project
Multiple Towers and Division Stories
Grade 4

NAME: _____ DATE: _____

- **BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.**
- **Show all work (diagrams, tables, and computations) on your answer sheet.**
- **If you do the work in your head, explain in writing how you did the work.**

The 17 tokens shown below are used to play a game. Each kind of token is worth a different number of points.



The picture below shows two clues about the number of points each kind of token is worth.

A. If each is worth 20 points, how many points is each worth? Show your work or explain how you got your answer.

B. If 4 friends play the game, they will divide the 17 tokens into 4 groups so that each friend's group of tokens is worth the same number of points. What is the total number of points each friend's group of tokens is worth? Show your work or explain how you got your answer.

2004 MCAS
Grade 4 Mathematics
Question 27 - Score Point 4

(a) ☆ = ⊙ ⊙
⊙ = ♡ ♡
⊙ ⊙ = ♡ ♡ ♡ ♡
☆ = ♡ ♡ ♡ ♡

Each ♡ is worth 5 points.

$4\overline{)20}$ = 5 points
20
0

☆ = 20 points

(b)
☆ = 20 points
♡ = 5 points
⊙ = 10 points

5 + 5 = 10
♡ ♡ = ⊙

6☆ = 6 × 20 = 120 points
6♡ = 5 × 6 = 30 points
5⊙ = 5 × 10 = +50 points
200 points

20
× 6
120

The total value of each group will be 50 points. To solve this problem I first found that ☆ = 20 points, ♡ = 5 points, and that ⊙ = 10 points. There are 6☆ so 6 × 20 = 120 points. There are 5 ⊙ so 5 × 10 = 50 points. There are 6♡ so 5 × 6 = 30 points. 120 + 50 + 30 = 200 points. Since there are 4 people 200 ÷ 4 = 50 points each.

$4\overline{)200}$ = 50
-20
00
-0
0

2004 MCAS
Grade 4 Mathematics
Question 27 - Score Point 3

a. If eatch ☆ 10
is worth 10 ◎ ◎ 10
and ◎ 10 is worth
5 ♡ 5 ♡, or ♡ ♡ ♡ ♡
that meens eatch ♡
is worth 5 points.

b. 50 points eatch

2004 MCAS
Grade 4 Mathematics
Question 27 - Score Point 2

A. I think that ♥ is worth 5 points. I found this out by looking at the cluse. If stare = 20 that means that the cricles are worth 10 points. So if one of the circle is equel to 2 ♥ that means that the to hearts have to be less than ten alone. But put together they have to make ten. So five is less than ten but put with another five equels 10! That is how I figured it out.

B. I think that the total value of each 4 players tokens will add up to: 35 points. I figured this out by naming the points of each token when one player gets it. To be divided up evinly each player will get one stare one circle and one heart. The points of each token 20, 10, and 5, plused together equel 35 points. That is how I figured it out.

$$\begin{array}{r} 10 \\ 20 \\ 5 \\ \hline 35 \end{array}$$

2004 MCAS
Grade 4 Mathematics
Question 27 - Score Point 1

(a) one heart token is worth 5 points

(b) each of the four groups gets 8 point - tokens because 8 * 4 = the number of points for the group of tokens

On-Demand Assessments

(To be filed in portfolio)

Multiple Towers and Division Stories

In class individualized On-Demand tasks assess knowledge of mathematical facts, operations, concepts, and skills, and their efficient application to problem solving. The results of these different forms of assessment provide rich profiles of students' achievements in mathematics and serve as the basis for identifying curricula and instructional approaches to best develop their talents.

UNIT: MULTIPLE TOWERS AND DIVISION STORIES

On-Demand Assessments

GRADE: 4

On-Demand Assessments (P) Multiple Towers and Division Stories In class individualized On-Demand tasks assess knowledge of mathematical facts, operations, concepts, and skills, and their efficient application to problem solving. The results of these different forms of assessment provide rich profiles of students' achievements in mathematics and serve as the basis for identifying curricula and instructional approaches to best develop their talents.	***Inv. :1*** Resource Binder, Session 1.5, M43 ***Inv. 2:*** Resource Binder, Session 2.6, M46* ***Inv. 3:*** Resource Binder, Session 3.4, M50* ***Inv. 4:*** Resource Binder, Session 4.5, M51 & M52* ***Please refer to the section in the Teacher's Unit Guide entitled, "Professional Development" for examples of student work for each assessment.**

Holyoke Public Schools
2007 - 2008
Mathematics
Scoring Rubric

Score point 4:
The response shows a ***comprehensive*** understanding of the mathematical concept(s) and/or procedures embodied in the task(s). It indicates that the student has ***completed the task(s) correctly***, using mathematically sound procedures. It contains ***clear, complete explanations*** and/or ***adequate work required***.

Score point 3:
The response shows a ***general*** understanding of the mathematical concept(s) and/or procedures embodied in the task(s). It indicates that the student has ***completed the task(s)***, using mathematically sound procedures. It contains ***complete explanations*** and/or ***adequate work required***.

Score point 2:
The response shows a ***basic*** understanding of the mathematical concept(s) and/or procedures embodied in the task(s). It addresses ***most aspects of the task(s)***, using mathematically sound procedures. It may contain a correct solution but provides ***incomplete procedures, reasoning and/or explanations***. It may reflect ***some misunderstandings*** of the underlying mathematical concepts and/or procedures.

Score point 1:
The response shows a ***minimal*** understanding of the mathematical concepts and/or procedures embodied in the task(s). It addresses ***some elements of the task(s) correctly*** but reaches an ***inadequate solution and/or provides reasoning that is faulty or incomplete***. It exhibits ***multiple flaws related to a misunderstanding of important aspects*** of the task(s), ***misuse*** of mathematical procedures, or faulty mathematical reasoning. It reflects a ***lack of essential understanding*** of the underlying mathematical concepts. It may contain a correct numerical answer but the ***required work is not provided***.

Score point 0:
The response is ***completely incorrect, irrelevant, or incoherent***, or contains a correct response arrived at using an ***obviously incorrect procedure***.

NOTES