



# Holyoke Public Schools

Grade 8

Growing, Growing, Growing

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# Curriculum Maps Outline

## **Map Goals:**

1. To ensure that students are exposed to a rigorous curriculum in every school and every grade.
2. To have consistent instruction and assessment district wide.
3. To prepare students for the MCAS test.
4. To explain what is expected to be covered in each CMP or Investigations Unit.

## **Expectations:**

The district's expectation is for students to successfully meet the Massachusetts Mathematics Standards. In order to help facilitate this, teachers are required to follow the curriculum maps. The successful implementation of these maps requires teachers to thoroughly read each lesson in the TE and work through the project and problems in the map and the text prior to planning their lessons. Work should be kept in the binder with the curriculum map. Working through the math is an essential part of lesson planning, as it helps the teacher to better understand the concept being taught and the students' possible misunderstandings.

## **Feedback To Students:**

Feedback needs to happen daily in the classroom. There are many ways to give feedback. Conferencing, observations, questions asked during your opening, work time and closing are all forms of feedback.

## **Map Components:**

1. GENERAL PROBING QUESTIONS
2. UNIT SPECIFIC PROBING QUESTIONS
3. GOALS OF UNIT, CONTENT STANDARDS, & PERFORMANCE STANDARDS
4. PROJECT- to be done at end of unit and kept in the portfolio.
5. STUDENT MASTER – for project
6. INVESTIGATIONS
7. NOTEBOOK - includes: 3 Ring Binder, Bound Notebook, Portfolio
8. ACCOUNTABLE TALK – using probing questions
9. ON DEMAND ASSESSMENTS - to be done during teaching of unit.
10. STUDENT MASTERS- for on-demand assessments.

## Mathematics Evidence of Learning Artifacts

Artifact	K - 1	2 – 5	6 - 8
<b>3 Ring Binder</b>  <b>(3R)*</b>	<ul style="list-style-type: none"> <li>○ Student Work<sup>1</sup></li> </ul>	<ul style="list-style-type: none"> <li>○ Vocabulary</li> <li>○ Student sheets<sup>1</sup></li> </ul> <p style="text-align: center;"><b><u>All work should be dated and listed by investigation</u></b></p>	<ul style="list-style-type: none"> <li>○ Math books</li> <li>○ Vocabulary</li> <li>○ Core Problems<sup>1</sup></li> <li>○ Lab sheets</li> </ul> <p style="text-align: center;"><b><u>All work should be dated and listed by investigation</u></b></p>
<b>Marble Notebook</b>  <b>(MNB)</b>	<ul style="list-style-type: none"> <li>○ Journal entries<sup>2</sup></li> </ul>	<ul style="list-style-type: none"> <li>○ Table of Contents</li> <li>○ Problem of the day</li> <li>○ Journal entries</li> <li>○ Class work</li> </ul> <p style="text-align: center;"><b><u>All work should be dated and listed by investigation in the Table of Contents</u></b></p>	<ul style="list-style-type: none"> <li>○ Table of Contents</li> <li>○ Work time</li> <li>○ Journal entries</li> </ul> <p style="text-align: center;"><b><u>All work should be dated and listed by investigation in the Table of Contents</u></b></p>
<b>Portfolio<sup>3</sup></b>  <b>(P)</b>	<ul style="list-style-type: none"> <li>○ On-demand tasks</li> <li>○ Projects</li> <li>○ Teacher anecdotal notes</li> </ul>	<ul style="list-style-type: none"> <li>○ On-demand tasks</li> <li>○ Reflections</li> <li>○ Projects</li> </ul> <p style="text-align: center;"><b><u>All work should be dated and listed by investigation</u></b></p>	<ul style="list-style-type: none"> <li>○ On-demand tasks</li> <li>○ Reflections</li> <li>○ Projects</li> </ul> <p style="text-align: center;"><b><u>All work should be dated and listed by investigation</u></b></p>

\* Folders may be used in place of binders for these grade levels

<sup>1</sup> Send home at the end of each unit

<sup>2</sup> Use grade level math journals

<sup>3</sup> All documents should be kept for the entire year

# Accountable Talk

## Probing Assessment Questions

As students progress through this unit, they should be asked the following questions to assess their specific knowledge of the unit.

- *What are the variables?*
- *Is the relationship between variables an example of exponential growth?*
- *How can the relationship be detected in a table, graph, or equation? What is the growth factor?*
- *What equation models the data in the table?*
- *What equation models the pattern in the graph?*
- *What can I learn about this situation by studying a table, graph, or equation of the exponential relationship?*
- *How does the relationship compare to other types of relationships that I have studied?*
- *Insert bullets of unit specific assessment questions to use during accountable talk*

## Probing Questions – Teacher’s Role

The teacher’s role in probing for understanding is to ask questions that will:

- *Clarify student understanding*
- *Get at the objective of the lesson*
- *Go deeper into the mathematics*
- *Uncover misconceptions and misunderstandings*
- *Compare and contrast*

## **Probing Questions – Student’s Role**

The students’ role is to be an active participant by:

- *Explaining their strategies*
- *Asking clarifying questions to teacher and other students*
- *Being active listeners*
- *Using the language of mathematics*

## **Probing Questions - Suggestions**

When probing for understanding the teacher and students can use one or more of these suggested questions:

- *Why are you using  $\langle \ \ \rangle$ ?*
- *What are the ways you could  $\langle \ \ \rangle$ ?*
- *What else do you know?*
- *How do you know that?*
- *Can you show that?*
- *What convention did you use here?*
- *What can you do if you do not know?*
- *What standard does this work apply to?*
- *Is this always true?*
- *How does this connect to other mathematics we have learned?*
- *What is the same and what are the differences between  $\langle \ \ \rangle$ ?*
- *Can you back that up?*
- *Where is the math in your sketch?*
- *What does the answer mean?*
- *Does the answer make sense?*
- *Could you have used another operation to solve this task?*
- *Can you give examples?*
- *Can you say it another way?*

- *What's the math?*
- *Tell me about the task in your own words?*
- *What are you trying to find?*
- *How did you make your estimate?*
- *Will your answer be an over-estimate or an under-estimate? Why?*
- *I noticed that you used <...> to help you understand the task. Can you show us what you did and tell us how it helped you?*
- *Where do you see < > in your <model, diagram, number line, chart, etc.>?*
- *How can we see < > in your <model, diagram, number line, chart, etc.>?*
- *You have used a representation that is different from others that I've seen. Can you show us your <model, diagram, number line, chart, etc.>, and tell us how it helped you?*
- *How did you decide to solve the task? Why did you choose that method?*
- *Did you try any method that didn't work?*
- *Tell us what you tried.*
- *Why didn't it work?*
- *Would it ever work? Insert bulleted list of unit specific questions for teacher and student to use during accountable talk*

# Goals, Content Standards, & Performance Standards

## Unit Goals:

- *Recognize situations in which one variable is an exponential function of another variable*
- *Recognize the connections between exponential equations and growth patterns in tables and graphs of those equations*
- *Construct equations to express exponential patterns that appear in data tables, graphs, and problem conditions*
- *Understand and apply the rules for operating on numerical expressions with exponents*
- *Solve problems about exponential growth from a variety of different subject areas, including science and business*
- *Compare exponential and linear relationships*

## Math Content Standards:

- *(8.N.7) Apply the rules of powers and roots to the solution of problems. Extend the Order of Operations to include positive integer exponents and square roots.*
- *(8.N.12) Select and use appropriate operations—addition, subtraction, multiplication, division, and positive integer exponents—to solve problems with rational numbers (including negatives).*
- *(8.P.1) Extend, represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic expressions. Include arithmetic and geometric progressions, e.g., compounding.*
- *(8.P.10) Use tables and graphs to represent and compare linear growth patterns. In particular, compare rates of change and  $x$ - and  $y$ -intercepts of different linear patterns.*

## Performance Standards:

- *(M3a) Discover, describe, and generalize patterns.*
- *(M3b) Represent relationships with tables, graphs, and verbal or symbolic rules.*
- *(M3b) Find solutions for unknown quantities in linear equations and in simple equations and inequalities.*

# Investigation 1: Exponential Growth

<p><b><u>Objectives</u></b> Investigations 1.1 -1.4</p>	<p><b><u>Pacing:</u></b> 5 days</p>
<p style="text-align: center;"><b><i>Evidence of Learning Artifacts</i></b></p> <p>Journal and Reflection questions should be posted and referred to at the beginning of the appropriate <i>Investigation</i>.</p> <p>Journal and Reflection entries need to be done in class as part of the closure and assessment</p>	<p><b><u>Vocabulary</u><sup>2</sup>:</b> <a href="#">Appendix 2</a>, Investigation 1</p> <hr/> <p><b><u>Core Problems</u><sup>2</sup>:</b> Growing, Growing, Growing, Investigation 1 ACE Problems: #2-4, 10-11, 15-21, 39-42; 22,23,47; 25-30</p> <hr/> <p><b><u>Work Time</u><sup>1</sup>:</b> Growing, Growing, Growing, Problems 1.1-1.4</p> <hr/> <p><b><u>Journal Entries</u><sup>1</sup>:</b> <a href="#">Appendix 3</a>, Inv 1.1-1.4</p> <hr/> <p><b><u>On Demand Tasks</u><sup>3</sup>:</b> <a href="#">Appendix 5</a>, Investigation 1</p> <hr/> <p><b><u>Mathematical Reflection</u><sup>3</sup></b> <a href="#">Appendix 4</a>, MMR1:</p>
<p style="text-align: center;"><b><i>Accountable Talk</i></b></p> <p>To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.</p>	<p>As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:</p> <ul style="list-style-type: none"> <li>○ How did you know that?</li> <li>○ How can you use ...?</li> <li>○ Can you show another way?</li> <li>○ What convention did you use?</li> </ul> <p>These are some recommended questions that you might use. Others can be found at the beginning of the map and on the probing question sheet in the district mathematics guide.</p>

1. Marble Note Book  
2.3-Ring Binder  
3. Portfolio

## Investigation 2: Examining Growth Patterns

<p><b><u>Objectives</u></b> Investigations 2.1-2.3</p>	<p><b><u>Pacing:</u></b> 4 days</p>
<p style="text-align: center;"><b><i>Evidence of Learning Artifacts</i></b></p> <p>Journal and Reflection questions should be posted and referred to at the beginning of the appropriate <i>Investigation</i>.</p> <p>Journal and Reflection entries need to be done in class as part of the closure and assessment</p>	<p><b><u>Vocabulary</u><sup>2</sup>:</b> <a href="#">Appendix 2</a>, Investigation 2</p> <p><b><u>Core Problems</u><sup>2</sup>:</b> Growing, Growing, Growing , Investigation 2 ACE Problems # 1, 2, 4, 15-16, 21; 5, 6, 8; 9-13, 24-27</p> <p><b><u>Work Time</u><sup>1</sup>:</b> Growing, Growing, Growing, Problems 2.1 - 2.3</p> <p><b><u>Journal Entries</u><sup>1</sup>:</b> <a href="#">Appendix 3</a>, Inv 2.1 - 2.3</p> <p><b><u>On Demand Tasks</u><sup>3</sup>:</b> <a href="#">Appendix 5</a>, Investigation 2</p> <p><b><u>Mathematical Reflection</u><sup>3</sup></b> <a href="#">Appendix 4</a>, MMR2:</p>
<p style="text-align: center;"><b><i>Accountable Talk</i></b></p> <p>To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.</p>	<p>As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:</p> <ul style="list-style-type: none"> <li>○ How did you know that?</li> <li>○ How can you use ...?</li> <li>○ Can you show another way?</li> <li>○ What convention did you use?</li> </ul> <p>These are some recommended questions that you might use. Others can be found at the beginning of the map and on the probing question sheet in the district mathematics guide.</p>

*1. Marble Note Book*

*2.3- Ring Binder*

*3. Portfolio*

*1. Marble Note Book*

*2.3- Ring Binder*

*3. Portfolio*

# Appendix 1 Unit Project

<p style="text-align: center;"><b><i>Project</i><sup>1</sup></b></p> <p>Student work should be placed in portfolio</p>	<p>The project is the culminating assessment, which will allow students to apply what they learned about the use of algebraic notation and reasoning. It is written in MCAS form to give students the experience of answering an open-response question. (From: <b>Check – Up #2 p 34</b>)</p> <p>The unit project is called ‘Exercise Program’ and the student handout for the project can be found in <a href="#">Appendix 1</a></p>
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*1. portfolio*

## Unit Project Scoring Guide

Score	Description
<u>4</u>	The student response demonstrates an exemplary understanding of the Patterns, Relations, and Algebra concepts involved in the use of tables to represent linear growth patterns. Given a verbal description, the student completes a table of values, analyzes the sequential pattern in order to create an equation, and then uses the equation to predict a real-world outcome.
<u>3</u>	The student response demonstrates a good understanding of the Patterns, Relations, and Algebra concepts involved in the use of tables to represent linear growth patterns. Although there is significant evidence that the student recognizes and applies the concepts involved, some aspect of the response is flawed. As a result, the response merits 3 points.
<u>2</u>	The student response demonstrates a fair understanding of the Patterns, Relations, and Algebra concepts involved in the use of tables to represent linear growth patterns. While some aspects of the task are completed correctly, others are not. The mixed evidence provided by the student merits 2 points.
<u>1</u>	The student response demonstrates a minimal understanding of the Patterns, Relations, and Algebra concepts involved in the use of tables to represent linear growth patterns.
<u>0</u>	The student response contains insufficient evidence of an understanding of the Patterns, Relations, and Algebra concepts involved in the use of tables to represent linear growth patterns to merit any points.

NAME: \_\_\_\_\_

DATE: \_\_\_\_\_

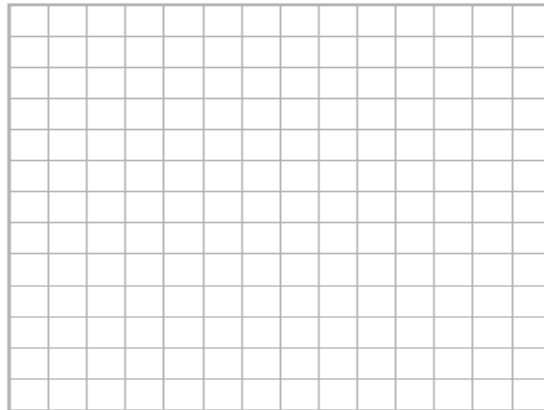
### Pumpkin Leaves

- BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.
- Show all work (diagrams, tables, and computations) on your answer sheet.
- If you do the work in your head, explain in writing how you did the work.

On the first day of school, you notice a few patches of fungus on the leaves of the pumpkin vines in your garden. You estimate the area covered by the fungus and find that the patches cover about  $1 \text{ cm}^2$ . Suppose that the leaf area covered by this kind of fungus quadruples every day.

- a. Complete the table to show what will happen to the fungus on the leaves during the first week of school.

Day	Area of Fungus ( $\text{cm}^2$ )
1	
2	
3	
4	
5	



- b. On the grid above, graph the (*day*, *area of fungus*) data from the table.
- c. Write an equation for the area of fungus  $A$  after  $d$  days.
- d. How much area will the fungus cover after 8 days? Explain how you found your answer.
- e. After how many days will the fungus cover at least  $1,000,000 \text{ cm}^2$ ?

## Appendix 2 Vocabulary

### **Investigation 1:**

- *base, pattern, relation, exponent, doubling, tripling, quadrupling, non-linear, linear relation, exponential form, standard form, exponential growth, growth factor, exponential relationship, scientific notation*

### **Investigation 2:**

- *none*

# Appendix 3 Journal Entries

## **Investigation 1:**

### ***Investigation 1.1:***

How is the number of ballots obtained with each cut related to the number of ballots before the cut?

### ***Investigation 1.2:***

Explain in what ways the chessboard and ballot-cutting situations are similar? In what ways are the two situations different? Use any new math vocabulary where appropriate.

### ***Investigation 1.3:***

Explain how the growth factor affects the shape of the graph?

### ***Investigation 1.4:***

In comparing Plans 1 and 4, explain what causes the difference in their patterns of change? Use appropriate math vocabulary

## **Investigation 2:**

### ***Investigation 2.1***

Explain at least two strategies to use in finding the number of months it would take the plant to completely cover the surface of the lake?

### ***Investigation 2.2***

What does each part of your equation tell you about the growth of the mold?

### ***Investigation 2.3***

Explain your strategy for writing the equation for the snake population.

## Appendix 4 Reflections

### MMR1

How is the number of ballots obtained with each cut related to the number of ballots before the cut?

### MMR2

Explain how you can use a table, a graph, and an equation to find the y-intercept and growth factor for an exponential relationship? Explain how you can use the y-intercept and growth factor to write an equation for an exponential relationship? [pg. 32 #1]

# Appendix 5 On Demand Tasks

## CMP2: Growing, Growing, Growing

<p><b><i>On-Demand Tasks</i></b></p> <p><b><u>Additional Practice &amp; Skills</u></b></p> <p><b><u>Workbook</u></b></p> <p><b><u>Assessment Resources</u></b></p> <p>In class individualized On-Demand tasks assess knowledge of mathematical facts, operations, concepts, and skills, and their efficient application to problem solving. The results of these different forms of assessment provide rich profiles of students' achievements in mathematics and serve as the basis for identifying curricula and instructional approaches to best develop their talents.</p>	<p><a href="#"><u>After Inv. 1</u></a> [Assessment Resource: p52 "Question Bank" #11a]</p> <p><a href="#"><u>After Inv. 2</u></a> [AP &amp; S pg.53 # 4]</p> <p>NOTE: typographical error in answer key <math>y=500(1.1)^x</math></p>
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## HPS Mathematics Scoring Rubric

Score	Description
<u>4</u>	The response shows a <b><u>comprehensive</u></b> understanding of the mathematical concept(s) and/or procedures embodied in the task(s). It indicates that the student has <b><u>completed the task(s)</u></b> correctly, using mathematically sound procedures. It contains <b><u>clear, complete explanations</u></b> and/or <b><u>adequate work required</u></b> .
<u>3</u>	The response shows a <b><u>general</u></b> understanding of the mathematical concept(s) and/or procedures embodied in the task(s). It indicates that the student has <b><u>completed the task(s)</u></b> , using mathematically sound procedures. It contains <b><u>complete explanations</u></b> and/or <b><u>adequate work required</u></b> .
<u>2</u>	The response shows a <b><u>basic</u></b> understanding of the mathematical concept(s) and/or procedures embodied in the task(s). It addresses <b><u>most aspects of the task(s)</u></b> , using mathematically sound procedures. It may contain a correct solution but provides <b><u>incomplete procedures, reasoning and/or explanations</u></b> . It may reflect <b><u>some misunderstandings</u></b> of the underlying mathematical concepts and/or procedures.
<u>1</u>	The response shows a <b><u>minimal</u></b> understanding of the mathematical concepts and/or procedures embodied in the task(s). It addresses <b><u>some elements of the task(s)</u></b> correctly but reaches an <b><u>inadequate solution and/or provides reasoning that is faulty or incomplete</u></b> . It exhibits <b><u>multiple flaws related to a misunderstanding of important aspects</u></b> of the task(s), misuse of mathematical procedures, or faulty mathematical reasoning. It reflects a <b><u>lack of essential understanding</u></b> of the underlying mathematical concepts. It may contain a correct numerical answer but <b><u>the required work is not provided</u></b> .
<u>0</u>	The response is <b><u>completely incorrect, irrelevant, or incoherent</u></b> , or contains a correct response arrived at using an <b><u>obviously incorrect procedure</u></b> .

NAME: \_\_\_\_\_

DATE: \_\_\_\_\_

## **Investigation 1**

- *BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.*
- *Show all work (diagrams, tables, and computations) on your answer sheet.*
- *If you do the work in your head, explain in writing how you did the work.*

Often when you try to learn new vocabulary words, you find that after a few days you have forgotten some of what you learned. Suppose you cram for a big test and memorize 100 new words and, for each day after the test, you forget 10% of the words you learned.

- a. Make a table and sketch a graph for the equation  $y = 3^x$ .

**NAME:**

**DATE:**

**Investigation 2**

- *BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.*
- *Show all work (diagrams, tables, and computations) on your answer sheet.*
- *If you do the work in your head, explain in writing how you did the work.*

**For Exercises 3–6, tell whether the relationship between  $x$  and  $y$  is linear, inverse, exponential, or neither, and explain your answer. If the relationship is linear, inverse, or exponential, write an equation for the relationship.**

4.

$x$	0	1	2	3	4	5
$y$	500	550	605	665.5	732.05	805.255

# NOTES