



# Holyoke Public Schools

## Grade 8

### Kaleidoscopes, Hubcaps, and Mirrors

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## **CURRICULUM MAPS OUTLINE**

### **Map Goals:**

1. To ensure that students are exposed to a rigorous curriculum in every school and every grade.
2. To have consistent instruction and assessment district wide.
3. To prepare students for the MCAS test.
4. To explain what is expected to be covered in each CMP or Investigations Unit.

### **Expectations:**

The district's expectation is for students to successfully meet the Massachusetts Mathematics Standards. In order to help facilitate this, teachers are required to follow the curriculum maps. The successful implementation of these maps requires teachers to thoroughly read each lesson in the TE and work through the project and problems in the map and the text prior to planning their lessons. Work should be kept in the binder with the curriculum map. Working through the math is an essential part of lesson planning, as it helps the teacher to better understand the concept being taught and the students' possible misunderstandings.

### **Feedback To Students:**

Feedback needs to happen daily in the classroom. There are many ways to give feedback. Conferencing, observations, questions asked during your opening, work time and closing are all forms of feedback.

### **Map Components:**

1. GENERAL PROBING QUESTIONS
2. UNIT SPECIFIC PROBING QUESTIONS
3. GOALS OF UNIT, CONTENT STANDARDS, & PERFORMANCE STANDARDS
4. PROJECT- to be done at end of unit and kept in the portfolio.
5. STUDENT MASTER – for project
6. INVESTIGATIONS
7. NOTEBOOK - includes: 3 Ring Binder, Bound Notebook, Portfolio
8. ACCOUNTABLE TALK – using probing questions
9. ON DEMAND ASSESSMENTS - to be done during teaching of unit.
10. STUDENT MASTERS- for on-demand assessments.

## Mathematics Evidence of Learning Artifacts

Artifact	K - 1	2 – 5	6 - 8
<b>3 Ring Binder</b>  <b>(3R)*</b>	<ul style="list-style-type: none"> <li>○ Student Work<sup>1</sup></li> </ul>	<ul style="list-style-type: none"> <li>○ Vocabulary</li> <li>○ Student sheets<sup>1</sup></li> </ul> <p style="text-align: center;"><b><u>All work should be dated and listed by investigation</u></b></p>	<ul style="list-style-type: none"> <li>○ Math books</li> <li>○ Vocabulary</li> <li>○ Core Problems<sup>1</sup></li> <li>○ Lab sheets</li> </ul> <p style="text-align: center;"><b><u>All work should be dated and listed by investigation</u></b></p>
<b>Marble Notebook</b>  <b>(MNB)</b>	<ul style="list-style-type: none"> <li>○ Journal entries<sup>2</sup></li> </ul>	<ul style="list-style-type: none"> <li>○ Table of Contents</li> <li>○ Problem of the day</li> <li>○ Journal entries</li> <li>○ Class work</li> </ul> <p style="text-align: center;"><b><u>All work should be dated and listed by investigation in the Table of Contents</u></b></p>	<ul style="list-style-type: none"> <li>○ Table of Contents</li> <li>○ Work time</li> <li>○ Journal entries</li> </ul> <p style="text-align: center;"><b><u>All work should be dated and listed by investigation in the Table of Contents</u></b></p>
<b>Portfolio<sup>3</sup></b>  <b>(P)</b>	<ul style="list-style-type: none"> <li>○ On-demand tasks</li> <li>○ Projects</li> <li>○ Teacher anecdotal notes</li> </ul>	<ul style="list-style-type: none"> <li>○ On-demand tasks</li> <li>○ Reflections</li> <li>○ Projects</li> </ul> <p style="text-align: center;"><b><u>All work should be dated and listed by investigation</u></b></p>	<ul style="list-style-type: none"> <li>○ On-demand tasks</li> <li>○ Reflections</li> <li>○ Projects</li> </ul> <p style="text-align: center;"><b><u>All work should be dated and listed by investigation</u></b></p>

\* Folders may be used in place of binders for these grade levels

<sup>1</sup> Send home at the end of each unit

<sup>2</sup> Use grade level math journals

<sup>3</sup> All documents should be kept for the entire year

# Accountable Talk

## Probing Assessment Questions

As students progress through this unit, they should be asked the following questions to assess their specific knowledge of the unit.

- *How can I use symmetry to describe the shapes and properties of figures in a design or a problem?*
- *Which figures in a pattern are congruent?*
- *What parts of a figure will be matched by a congruence transformation?*
- *How would you describe the steps used to transform a given figure to its image ?*

## Probing Questions – Teacher’s Role

The teacher’s role in probing for understanding is to ask questions that will:

- *Clarify student understanding*
- *Get at the objective of the lesson*
- *Go deeper into the mathematics*
- *Uncover misconceptions and misunderstandings*
- *Compare and contrast*

## Probing Questions – Student’s Role

The students’ role is to be an active participant by:

- *Explaining their strategies*
- *Asking clarifying questions to teacher and other students*
- *Being active listeners*
- *Using the language of mathematics*

## Probing Questions - Suggestions

When probing for understanding the teacher and students can use one or more of these suggested questions:

- *Why are you using  $< >$ ?*
- *What are the ways you could  $< >$ ?*
- *What else do you know?*
- *How do you know that?*
- *Can you show that?*
- *What convention did you use here?*
- *What can you do if you do not know?*
- *What standard does this work apply to?*
- *Is this always true?*
- *How does this connect to other mathematics we have learned?*
- *What is the same and what are the differences between  $< >$ ?*
- *Can you back that up?*
- *Where is the math in your sketch?*
- *What does the answer mean?*
- *Does the answer make sense?*
- *Could you have used another operation to solve this task?*
- *Can you give examples?*
- *Can you say it another way?*
- *What's the math?*
- *Tell me about the task in your own words?*
- *What are you trying to find?*
- *How did you make your estimate?*
- *Will your answer be an over-estimate or an under-estimate? Why?*
- *I noticed that you used  $<....>$  to help you understand the task. Can you show us what you did and tell us how it helped you?*
- *Where do you see  $< >$  in your  $<model, diagram, number line, chart, etc.>$ ?*
- *How can we see  $< >$  in your  $<model, diagram, number line, chart, etc.>$ ?*

- *You have used a representation that is different from others that I've seen. Can you show us your <model, diagram, number line, chart, etc.>, and tell us how it helped you?*
- *How did you decide to solve the task? Why did you choose that method?*
- *Did you try any method that didn't work?*
- *Tell us what you tried.*
- *Why didn't it work?*
- *Would it ever work?*

# Goals, Content Standards, & Performance Standards

## Unit Goals:

- *Understand important properties of symmetry*
- *Recognize and describe symmetries of figures*
- *Use tools to examine symmetries and transformations*
- *Make figures with specified symmetries*
- *Identify a basic design element that can be used with a transformation to replicate a given design*
- *Perform symmetry transformations of figures, including reflections, translations, and rotations*
- *Examine and describe the symmetries of a design made from a figure and its image under a symmetry transformation*
- *Give precise mathematical directions for performing reflections, rotations, and translations in terms of the effect of the transformation on points of the original figure*
- *Draw conclusions about a figure in terms of the effect of the transformation on points of the original figure based on what symmetry or symmetries the figure has*
- *Understand that figures with the same shape and size are congruent*
- *Use symmetry transformations to explore whether two figures are congruent*
- *Give examples of minimum sets of measures of angles and sides that will guarantee that two triangles are congruent*
- *Use congruence of triangles to explore congruence of two quadrilaterals*
- *Use symmetry and congruence to deduce properties of figure*
- *Write coordinate rules for specifying the image of a point under particular transformations*

## **Math Content Standards:**

- *(8.G.2.) Classify figures in terms of congruence and similarity, and apply these relationships to the solution of problems.*
- *(8.G.3) Demonstrate an understanding of the relationships of angles formed by intersecting lines, including parallel lines cut by a transversal.*
- *(8.G.5) Use a straight-edge, compass, or other tools to formulate and test conjectures, and to draw geometric figures.*
- *(8.G.6) Predict the results of transformations on unmarked or coordinate planes and draw the transformed figure, e.g., predict how tessellations transform under translations, reflections, and rotations.*
- *(8.M.4) Use ratio and proportion (including scale factors) in the solution of problems, including problems involving similar plane figures and indirect measurement.*

## **Performance Standards:**

- *(M2a) Is familiar with assorted two- and three-dimensional objects, including squares, triangles, other polygons, circles, cubes, rectangular prisms, pyramids, spheres, and cylinders.*
- *(M2b) Identifies similar and congruent shapes and uses transformations in the coordinate plane, i.e., transformations, rotations, and reflections.*
- *(M2e) Recognizes similarity and rotational and bilateral symmetry in two- and three-dimensional figures.*
- *(M2f) Analyzes and generalizes geometric patterns, such as tessellations and sequences of shapes.*

# Investigation 1: Three Types of Symmetry

<p><b><u>Objectives</u></b> Investigations Problems 1.1 – 1.4</p>	<p><b><u>Pacing:</u></b> 4 days</p>
<p style="text-align: center;"><b><i>Evidence of Learning Artifacts</i></b></p> <p>Journal and Reflection questions should be posted and referred to at the beginning of the appropriate <i>Investigation</i>.</p> <p>Journal and Reflection entries need to be done in class as part of the closure and assessment</p>	<p><b><u>Vocabulary</u><sup>2</sup>:</b> <a href="#">Appendix 2</a>, Investigation 1</p> <hr/> <p><b><u>Core Problems</u><sup>2</sup>:</b> Kaleidoscopes, Hubcaps, and Mirrors, Investigation 1 ACE Problems: #1-17, 20 – 30, 34-40, 46</p> <hr/> <p><b><u>Work Time</u><sup>1</sup>:</b> Kaleidoscopes, Hubcaps, and Mirrors, Problems 1.1 – 1.4</p> <hr/> <p><b><u>Journal Entries</u><sup>1</sup>:</b> <a href="#">Appendix 3</a>, Inv 1.1 – 1.4</p> <hr/> <p><b><u>On Demand Tasks</u><sup>3</sup>:</b> <a href="#">Appendix 5</a>, Investigation 1</p> <hr/> <p><b><u>Mathematical Reflection</u><sup>3</sup></b> <a href="#">Appendix 4</a>, MMR1:</p>
<p style="text-align: center;"><b><i>Accountable Talk</i></b></p> <p>To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.</p>	<p>As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:</p> <ul style="list-style-type: none"> <li>○ How did you know that?</li> <li>○ How can you use ...?</li> <li>○ Can you show another way?</li> <li>○ What convention did you use?</li> </ul> <p>These are some recommended questions that you might use. Others can be found at the beginning of the map and on the probing question sheet in the district mathematics guide.</p>

*1. Marble Note Book*  
*2.3 Ring Binder*  
*3. Portfolio*

## Investigation 2: Symmetry Transformations

<p><b>Objectives</b> Investigations 2.1 – 2.4</p>	<p><b>Pacing:</b> 5 days</p>
<p style="text-align: center;"><b><i>Evidence of Learning Artifacts</i></b></p> <p>Journal and Reflection questions should be posted and referred to at the beginning of the appropriate <i>Investigation</i>.</p> <p>Journal and Reflection entries need to be done in class as part of the closure and assessment</p>	<p><b><u>Vocabulary</u><sup>2</sup>:</b> <a href="#">Appendix 2</a>, Investigation 2</p> <hr/> <p><b><u>Core Problems</u><sup>2</sup>:</b> Kaleidoscopes, Hubcaps, and Mirrors, Investigation 2 ACE Problems: # 1-5, 16, 6,7,19,20, 8,9,21-23, 10-15</p> <hr/> <p><b><u>Work Time</u><sup>1</sup>:</b> Kaleidoscopes, Hubcaps, and Mirrors, Problems 2.1 – 2.4</p> <hr/> <p><b><u>Journal Entries</u><sup>1</sup>:</b> <a href="#">Appendix 3</a>, Inv 2.1 – 2.4</p> <hr/> <p><b><u>On Demand Tasks</u><sup>3</sup>:</b> <a href="#">Appendix 5</a>, Investigation 2</p> <hr/> <p><b><u>Mathematical Reflection</u><sup>3</sup></b> <a href="#">Appendix 4</a>, MMR2:</p>
<p style="text-align: center;"><b><i>Accountable Talk</i></b></p> <p>To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.</p>	<p>As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:</p> <ul style="list-style-type: none"> <li>○ How did you know that?</li> <li>○ How can you use ...?</li> <li>○ Can you show another way?</li> <li>○ What convention did you use?</li> </ul> <p>These are some recommended questions that you might use. Others can be found at the beginning of the map and on the probing question sheet in the district mathematics guide.</p>

*1. Marble Note Book*

*2.3-Ring Binder*

*3. Portfolio*

## Investigation 3: Exploring Congruence

<p><b><u>Objectives</u></b> Investigations 3.1 – 3.</p>	<p><b><u>Pacing:</u></b> 2 days</p>
<p style="text-align: center;"><b><i>Evidence of Learning Artifacts</i></b></p> <p>Journal and Reflection questions should be posted and referred to at the beginning of the appropriate <i>Investigation</i>.</p> <p>Journal and Reflection entries need to be done in class as part of the closure and assessment</p>	<p><b><u>Vocabulary</u></b><sup>2</sup>: <a href="#">Appendix 2</a>, Investigation 3</p> <p><b><u>Core Problems</u></b><sup>2</sup>: Kaleidoscopes, Hubcaps, and Mirrors, Investigation 3 ACE Problems: replace with pg and question #s</p> <p><b><u>Work Time</u></b><sup>1</sup>: Kaleidoscopes, Hubcaps, and Mirrors, Problems 3.1 – 3.2</p> <p><b><u>Journal Entries</u></b><sup>1</sup>: <a href="#">Appendix 3</a>, Inv 3.1 – 3.2</p> <p><b><u>On Demand Tasks</u></b><sup>3</sup>: <a href="#">Appendix 5</a>, Investigation 3</p> <p><b><u>Mathematical Reflection</u></b><sup>3</sup> <a href="#">Appendix 4</a>, MMR3:</p>
<p style="text-align: center;"><b><i>Accountable Talk</i></b></p> <p>To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.</p>	<p>As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:</p> <ul style="list-style-type: none"> <li>○ How did you know that?</li> <li>○ How can you use ...?</li> <li>○ Can you show another way?</li> <li>○ What convention did you use?</li> </ul> <p>These are some recommended questions that you might use. Others can be found at the beginning of the map and on the probing question sheet in the district mathematics guide.</p>

1. Marble Note Book

2.3- Ring Binder

3. Portfolio

## Investigation 4: Applying Congruence and Symmetry

<b>Objectives</b> Investigation 4.1 only	<b>Pacing:</b> 1 day
<p style="text-align: center;"><b><i>Evidence of Learning Artifacts</i></b></p> <p>Journal and Reflection questions should be posted and referred to at the beginning of the appropriate <i>Investigation</i>.</p> <p>Journal and Reflection entries need to be done in class as part of the closure and assessment</p>	<p><b><u>Vocabulary</u><sup>2</sup>:</b> <a href="#">Appendix 2</a>, Investigation 4</p> <p><b><u>Core Problems</u><sup>2</sup>:</b> Kaleidoscopes, Hubcaps, and Mirrors, Investigation 4 ACE Problems: #1 – 9, 16</p> <p><b><u>Work Time</u><sup>1</sup>:</b> Kaleidoscopes, Hubcaps, and Mirrors, Problem 4.1 only</p> <p><b><u>Journal Entries</u><sup>1</sup>:</b> <a href="#">Appendix 3</a>, Inv 4.1 only</p> <p><b><u>On Demand Tasks</u><sup>3</sup>:</b> <a href="#">Appendix 5</a>, Investigation 4</p> <p><b><u>Mathematical Reflection</u><sup>3</sup></b> <a href="#">Appendix 4</a>, MMR4:</p>
<p style="text-align: center;"><b><i>Accountable Talk</i></b></p> <p>To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.</p>	<p>As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:</p> <ul style="list-style-type: none"> <li>○ How did you know that?</li> <li>○ How can you use ...?</li> <li>○ Can you show another way?</li> <li>○ What convention did you use?</li> </ul> <p>These are some recommended questions that you might use. Others can be found at the beginning of the map and on the probing question sheet in the district mathematics guide.</p>

1. *Marble Note Book*
2. *3-Ring Binder*
3. *Portfolio*

## Investigation 5: Transforming Coordinates

<b>Objectives</b> Investigations 5.1 – 5.3	<b>Pacing:</b> 3 days
<p style="text-align: center;"><b><i>Evidence of Learning Artifacts</i></b></p> <p>Journal and Reflection questions should be posted and referred to at the beginning of the appropriate <i>Investigation</i>.</p> <p>Journal and Reflection entries need to be done in class as part of the closure and assessment</p>	<p><b><u>Vocabulary</u><sup>2</sup>:</b> <a href="#">Appendix 2</a>, Investigation 5</p>
	<p><b><u>Core Problems</u><sup>2</sup>:</b> Kaleidoscopes, Hubcaps, and Mirrors, Investigation 5 ACE Problems: #1-3, 4,20,21, 5-14,22</p>
	<p><b><u>Work Time</u><sup>1</sup>:</b> Kaleidoscopes, Hubcaps, and Mirrors, Problem 5.1-5.3</p>
	<p><b><u>Journal Entries</u><sup>1</sup>:</b> <a href="#">Appendix 3</a>, Inv 5.1 – 5.3</p>
	<p><b><u>On Demand Tasks</u><sup>3</sup>:</b> <a href="#">Appendix 5</a>, Investigation 5</p>
	<p><b><u>Mathematical Reflection</u><sup>3</sup></b> <a href="#">Appendix 4</a>, MMR5:</p>
<p style="text-align: center;"><b><i>Accountable Talk</i></b></p> <p>To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.</p>	<p>As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:</p> <ul style="list-style-type: none"> <li>○ How did you know that?</li> <li>○ How can you use ...?</li> <li>○ Can you show another way?</li> <li>○ What convention did you use?</li> </ul> <p>These are some recommended questions that you might use. Others can be found at the beginning of the map and on the probing question sheet in the district mathematics guide.</p>

1. *Marble Note Book*  
2.3-*Ring Binder*  
3. *Portfolio*

# Appendix 1 Unit Project

<p><b><i>Project</i><sup>1</sup></b></p> <p>Student work should be placed in portfolio</p>	<p>The project is the culminating assessment, which will allow students to apply what they learned about symmetry and transformations. It is written in MCAS form to give students the experience of answering an open-response question.</p> <p>The unit project is called “Chelsea’s Triangle” and the student handout for the project can be found in <a href="#">Appendix 1</a></p>
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*1. portfolio*

## Unit Project Scoring Guide

	Description
<b><u>4</u></b>	The student response demonstrates an exemplary understanding of the Geometry concepts involved in predicting the results of reflections of a figure on a coordinate plane and drawing the transformed image. The student performs a reflection of a geometric shape followed by another reflection of the resulting image, then labels and identifies the coordinate locations for all corresponding image vertices.
<b><u>3</u></b>	The student response demonstrates a good understanding of the Geometry concepts involved in predicting the results of reflections of a figure on a coordinate plane and drawing the transformed image. Although there is significant evidence that the student recognizes and applies the concepts involved, some aspect of the response is flawed. As a result, the response merits 3 points.
<b><u>2</u></b>	The student response demonstrates a fair understanding of the Geometry concepts involved in predicting the results of reflections of a figure on a coordinate plane and drawing the transformed image. While some aspects of the task are completed correctly, others are not. The mixed evidence provided by the student merits 2 points.
<b><u>1</u></b>	The student response demonstrates a minimal understanding of the Geometry concepts involved in predicting the results of reflections of a figure on a coordinate plane and drawing the transformed image.
<b><u>0</u></b>	The student response contains insufficient evidence of an understanding of the Geometry concepts involved in predicting the results of reflections of a figure on a coordinate plane and drawing the transformed image to merit any points.

NAME: \_\_\_\_\_

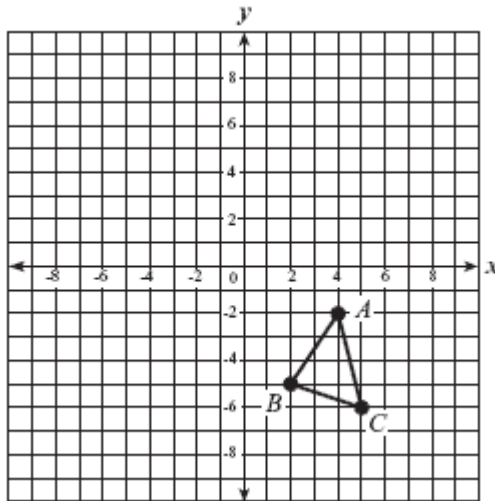
DATE: \_\_\_\_\_

### Chelsea's Triangle

- BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.
- Show all work (diagrams, tables, and computations) on your answer sheet.
- If you do the work in your head, explain in writing how you did the work.

Standard(s): 8.G.6

Chelsea drew triangle  $ABC$  so that the vertices are at points  $A(4,-2)$ ,  $B(2,-5)$ , and  $C(5,-6)$ , as shown on the coordinate grid below.



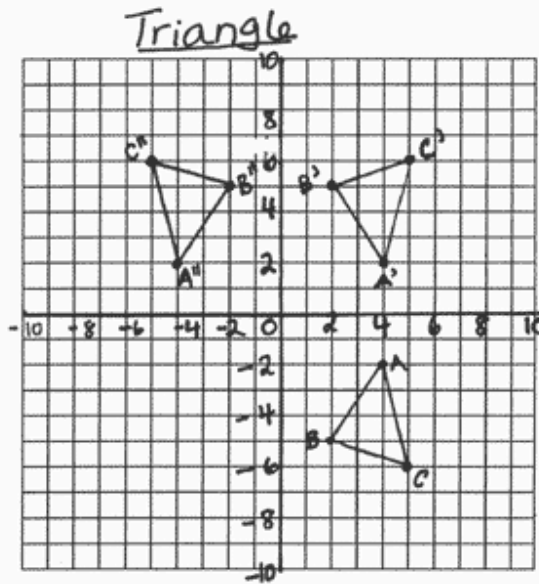
- Copy the coordinate grid and triangle  $ABC$  onto the grid in your Student Answer Booklet. Draw the reflection of triangle  $ABC$  across the  $x$ -axis to form triangle  $A'B'C'$ . List the coordinates for point  $A'$ , point  $B'$ , and point  $C'$ .
- On the same coordinate grid, draw the reflection of triangle  $A'B'C'$  across the  $y$ -axis to form triangle  $A''B''C''$ . List the coordinates for point  $A''$ , point  $B''$ , and point  $C''$ .

# 2006 MCAS Grade 8 Mathematics

## Student Work: Question 22- Score Point 4

Ⓐ The coordinates for the points  $A'$ ,  $B'$ , and  $C'$  are:  
 $A'(4, 2)$ ,  $B'(2, 5)$ , and  $C'(5, 6)$ .

Ⓑ The coordinates for points  $A''$ ,  $B''$ , and  $C''$  are:  
 $A''(-4, 2)$ ,  $B''(-2, 5)$ , and  $C''(-5, 6)$ .

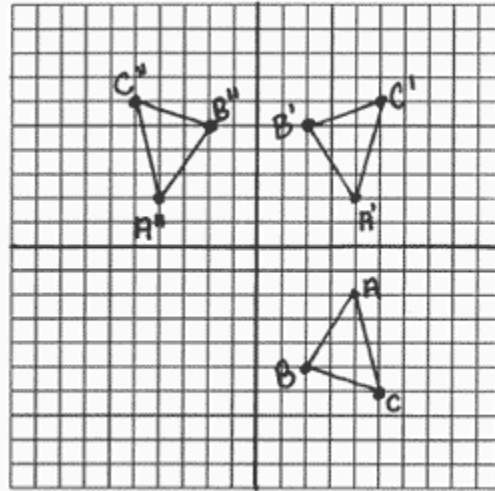


# 2006 MCAS Grade 8 Mathematics

## Student Work: Question 22 - Score Point 3

a) The coordinates of the  
Reflection would be  
 $A'(4,2)$   $B'(2,5)$   $C(5,6)$

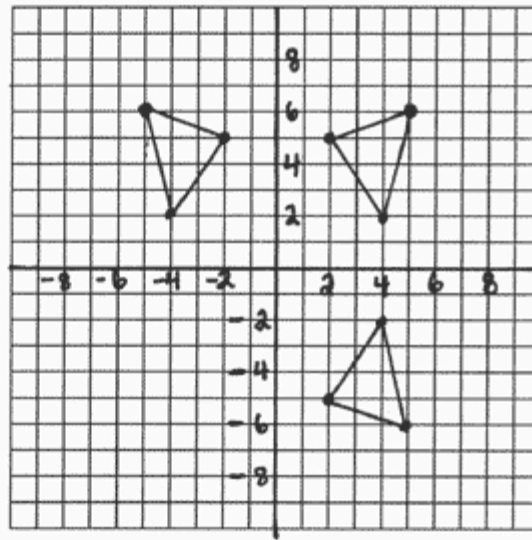
B) The coordinates  
would be  
 $A''(4,2)$   $B''(2,5)$   $C''(5,6)$



# 2006 MCAS Grade 8 Mathematics

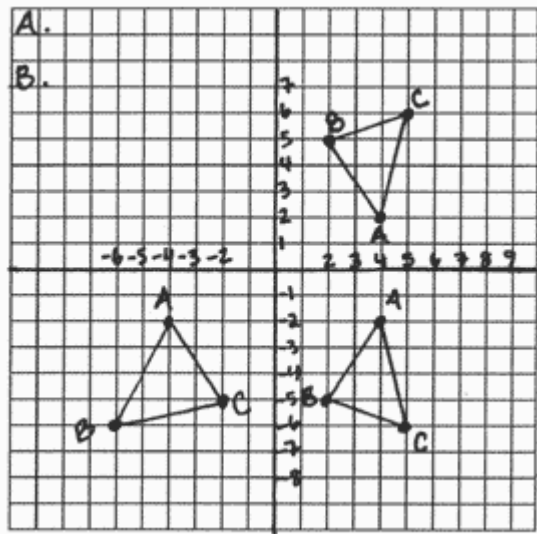
## Student Work: Question 22 - Score Point 2

a. graph  
b. graph



# 2006 MCAS Grade 8 Mathematics

## Student Work: Question 22 - Score Point 1



## Appendix 2 Vocabulary

### **Investigation 1:**

- *symmetry, line symmetry, line of symmetry, reflection symmetry, rotation, rotation symmetry, angle of rotation, center of rotation, basic design element, kaleidoscope, translation, translation symmetry, magnitude and direction of translation, translation image*

### **Investigation 2:**

- *rotations, symmetry, reflection, reflection symmetry, tessellation, basic design element, vertices (3R)*

### **Investigation 3:**

- *congruent, congruence, corresponds(3R)*

### **Investigation 4:**

- *none*

### **Investigation 5:**

- *coordinate plane, coordinate rules*

# Appendix 3 Journal Entries

## **Investigation 1:**

### ***Investigation 1.1:***

Describe and give examples of reflection symmetry.

### ***Investigation 1.2:***

Suppose you know the angle of rotation for a particular design. How can you use it to find all the other angles through which the design can be rotated to match the original design

### ***Investigation 1.3:***

What mathematical relationship do you see between the number of lines of symmetry and the angle of rotation?

### ***Investigation 1.4:***

How is a basic design element related to translation symmetry?

## **Investigation 2:**

### ***Investigation 2.1***

Describing Line Reflections: How does a vertex on a polygon correspond to a vertex on an image of that polygon?

### ***Investigation 2.2***

Describing Rotations: To rotate a copy of a figure, what two pieces of information do you need?

### ***Investigation 2.3***

Describing Translations: What is the relationship between a figure and its image under a translation?

### ***Investigation 2.4***

Using Symmetry to Think About Tessellations: What is a basic design element and how is it used to create a tessellation?

## **Investigation 3**

### ***Investigation 3.1***

Relating Symmetry and Congruence Describe how you use symmetry transformations to show that two figures are congruent.

### ***Investigation 3.2***

Congruent Triangles. Describe ways, by measuring some sides and angles, that you can check whether two triangles are congruent

## **Investigation 4**

### ***Investigation 4.1***

Finding Distances without Measuring Describe what it means for two triangles to be congruent. Describe what it means for two triangles to be similar?

## **Investigation 5**

### ***Investigation 5.1***

Coordinate Rules for Reflections What pattern did you notice, between the points of an image and its reflection?

### ***Investigation 5.2***

Coordinate Rules for Translations If you are given a rule, how can you recognize whether it describes a translation?

### ***Investigation 5.3***

Coordinate Rules for Rotations Specify coordinate rules for rotations of  $90^\circ$  and  $270^\circ$

# Appendix 4 Reflections

## MMR1

How would you explain to someone what it means for a figure to have:

- a. reflection symmetry?
- b. rotation symmetry?
- c. translation symmetry?

## MMR2

[pg. 47 #1,2,3]

## MMR3

What does it mean to say that two geometric figures are congruent?

## MMR4

[pg 77 #1c, #1d]

## MMR5

Select 2 symmetry transformations from the list on p.95. What is the image of a point  $(x,y)$  under each transformation?

# Appendix 5 On Demand Tasks

## CMP2: Kaleidoscopes, Hubcaps, and Mirrors

<p><b><i>On-Demand Tasks</i></b></p> <p><b><u><i>Additional Practice &amp; Skills Workbook</i></u></b></p> <p><b><u><i>Assessment Resources</i></u></b></p> <p>In class individualized On-Demand tasks assess knowledge of mathematical facts, operations, concepts, and skills, and their efficient application to problem solving. The results of these different forms of assessment provide rich profiles of students' achievements in mathematics and serve as the basis for identifying curricula and instructional approaches to best develop their talents.</p>	<p><a href="#"><u><i>After Inv. 1</i></u></a></p> <p><i>Additional Practice &amp; Skills Workbook</i> pg. 79 #1-4</p> <p><a href="#"><u><i>After Inv. 2</i></u></a></p> <p><i>Additional Practice &amp; Skills Workbook</i> pg 90 # 19</p> <p><a href="#"><u><i>After Inv. 3</i></u></a></p> <p><i>Additional Practice &amp; Skills Workbook</i> pg 95 #3</p> <p><a href="#"><u><i>After Inv. 4</i></u></a></p> <p><i>ACE QUESTION</i> pg72 #10</p> <p><a href="#"><u><i>After Inv. 5</i></u></a></p> <p><i>ACE QUESTION</i> pg.92 #24] Use with “Strategies”</p>
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## HPS Mathematics Scoring Rubric

Score	Description
<u>4</u>	The response shows a <b><u>comprehensive</u></b> understanding of the mathematical concept(s) and/or procedures embodied in the task(s). It indicates that the student has <b><u>completed the task(s)</u></b> correctly, using mathematically sound procedures. It contains <b><u>clear, complete explanations</u></b> and/or <b><u>adequate work required</u></b> .
<u>3</u>	The response shows a <b><u>general</u></b> understanding of the mathematical concept(s) and/or procedures embodied in the task(s). It indicates that the student has <b><u>completed the task(s)</u></b> , using mathematically sound procedures. It contains <b><u>complete explanations</u></b> and/or <b><u>adequate work required</u></b> .
<u>2</u>	The response shows a <b><u>basic</u></b> understanding of the mathematical concept(s) and/or procedures embodied in the task(s). It addresses <b><u>most aspects of the task(s)</u></b> , using mathematically sound procedures. It may contain a correct solution but provides <b><u>incomplete procedures, reasoning and/or explanations</u></b> . It may reflect <b><u>some misunderstandings</u></b> of the underlying mathematical concepts and/or procedures.
<u>1</u>	The response shows a <b><u>minimal</u></b> understanding of the mathematical concepts and/or procedures embodied in the task(s). It addresses <b><u>some elements of the task(s)</u></b> correctly but reaches an <b><u>inadequate solution and/or provides reasoning that is faulty or incomplete</u></b> . It exhibits <b><u>multiple flaws related to a misunderstanding of important aspects</u></b> of the task(s), misuse of mathematical procedures, or faulty mathematical reasoning. It reflects a <b><u>lack of essential understanding</u></b> of the underlying mathematical concepts. It may contain a correct numerical answer but <b><u>the required work is not provided</u></b> .
<u>0</u>	The response is <b><u>completely incorrect, irrelevant, or incoherent</u></b> , or contains a correct response arrived at using an <b><u>obviously incorrect procedure</u></b> .

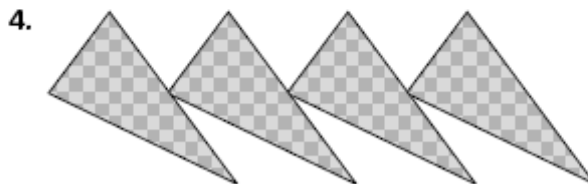
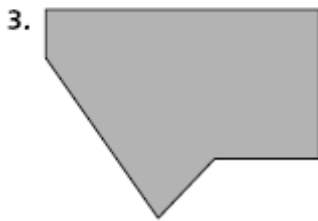
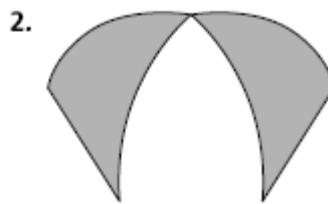
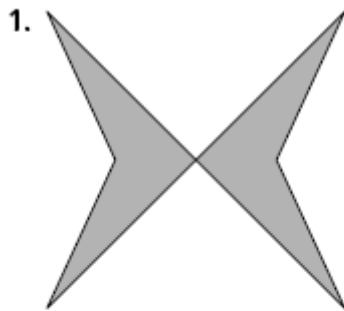
NAME: \_\_\_\_\_

DATE: \_\_\_\_\_

**Investigation 1**

- *BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.*
- *Show all work (diagrams, tables, and computations) on your answer sheet.*
- *If you do the work in your head, explain in writing how you did the work.*

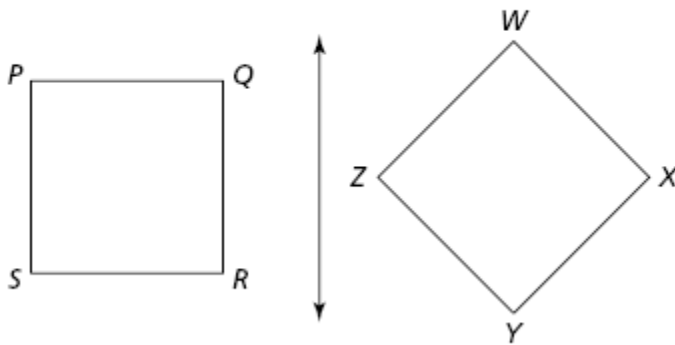
**Determine all the types of symmetry in the design. Specify lines of symmetry, centers and angles of rotation, and lengths and directions of translations.**



## Investigation 2

- BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.
- Show all work (diagrams, tables, and computations) on your answer sheet.
- If you do the work in your head, explain in writing how you did the work.

19. Describe two different sets of transformations that would move square  $PQRS$  onto square  $WXYZ$ .

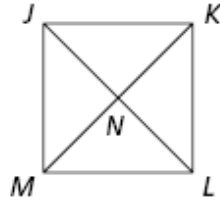


NAME: \_\_\_\_\_

DATE: \_\_\_\_\_

### Investigation 3

3. Use the figure of square  $JKLM$  below to answer (a) and (b).

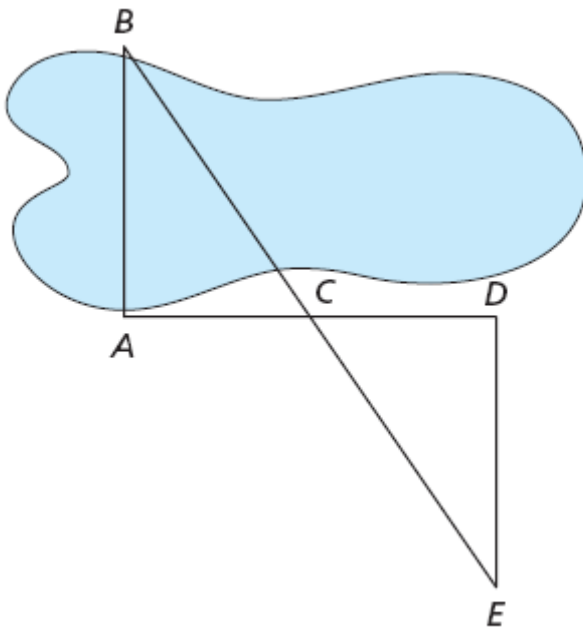


- a. List all triangles in the figure above which are congruent to triangle  $JNM$ . Explain.
- b. List all triangles congruent to triangle  $MKL$ . Explain.

**Investigation 4**

- *BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.*
- *Show all work (diagrams, tables, and computations) on your answer sheet.*
- *If you do the work in your head, explain in writing how you did the work.*

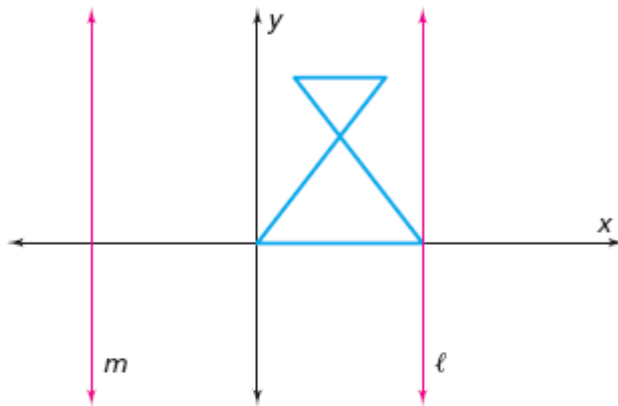
Alejandro wants to measure the distance directly across a pond from  $A$  to  $B$ . He uses string and some stakes to create the setup shown in the diagram below. None of his string can cross the pond. What information does Alejandro need to build into his setup to find the length of  $\overline{AB}$ ?



## Investigation 5

- BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.
- Show all work (diagrams, tables, and computations) on your answer sheet.
- If you do the work in your head, explain in writing how you did the work.

**Multiple Choice** Which of these statements is *not* true about the figure below?



- A.** Reflecting the figure in the  $y$ -axis, and then reflecting the image in the  $x$ -axis, gives the same final image as rotating the figure  $180^\circ$  about the origin.
- B.** Reflecting the figure in line  $\ell$ , and then reflecting the image in the  $y$ -axis, gives the same final image as reflecting the figure in line  $m$ .
- C.** Reflecting the figure in the  $y$ -axis and then rotating the image  $180^\circ$  about the origin gives the same final image as reflecting the figure in the  $x$ -axis.
- D.** Rotating the figure  $90^\circ$  counterclockwise about the origin and then rotating the image another  $90^\circ$  counterclockwise gives the same image as rotating the original image  $180^\circ$  about the origin.

# ▪ *PRACTICING MULTIPLE CHOICE QUESTIONS*

## **What teachers need to do**

- Make clear to all students that their job is not simply to select the correct answer. They must show their work AND say why at least one of the given wrong answers can distract students from selecting the correct answer.
- Stress that the purpose of this strategy is not to try to teach students to make better guesses on multiple-choice tests. The purpose is for students to learn more about the errors they frequently make and in the process to develop a deeper understanding of the math. That is a far more reliable kind of test preparation than learning how to guess.

## INFORMATION SHEET FOR STUDENTS: LEARNING FROM INCORRECT ANSWERS

### How this strategy works to help you prepare for the test

In each question you are to select one correct answer from four possible choices. Only *one* of these answers is the correct one. People who make up multiple-choice questions usually follow this pattern when they make up the test:

- One of the answers is the correct one.
- The other answers are wrong.
- *One* of the wrong answers is very close to the correct one. It is the one that you would probably choose if you made a common mistake.

Here is an example:

What is  $\frac{1}{2} + \frac{1}{3}$  ?

(A)  $\frac{2}{5}$  (B)  $\frac{5}{6}$  (C)  $\frac{1}{6}$  (D)  $\frac{1}{9}$

The correct choice is (B).

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

however, some students think that the correct answer is (A). It is NOT because:

$$\frac{1}{2} + \frac{1}{3} \text{ is NOT equal to } \frac{2}{5}$$

Students that select (A) probably do so because they forget that when you add fractions you cannot just add the numerator and the denominator. To add fractions you must find the common denominator first.

If you practice how to spot these common mistakes, you will be able to learn how to avoid them, and so *score higher* on the test.

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### **Rules for answering these questions**

1. Read the question carefully and think hard about it.
2. Write down all the work that you did to figure out the correct answer.
3. Look at the three wrong answers and ask yourself this question:  
“What common mistake could lead a student to select one of these wrong answers as the correct one.”
4. Write down the wrong answer that students might choose if they had made a mistake. Show that mistake step by step.

# NOTES